

Representing a Solid #1

(Function based representation)

Computer Vision

9-1

Classification of representation

- ◆ Surface based representation
 - represented as a collection of surfaces
 - » curvature primal sketch
 - » extended gaussian image
 - » aspect graph
 - » b-rep (winged-edge)
 - » well-tessellated surface
- ◆ Function based representation
 - represented as a function and its parameters
 - » generalized cylinder
 - » superquadric
 - » symmetry seeking
 - » spherical attribute image
- ◆ Volumetric based representation
 - represented as a collection of primitive solids
 - » constructive solid geometry
 - » occupancy graph
 - » oct-tree

Representing a Solid #1

(Function based representation)

1. Classification of representation
2. Superquadric
3. Generalized cylinder
4. Symmetry seeking
5. Spherical attribute image

Superquadric representation

◆ Generalization of ellipsoid

Ellipsoid $\left[\frac{x}{a}\right]^2 + \left[\frac{y}{b}\right]^2 + \left[\frac{z}{c}\right]^2 = 1$

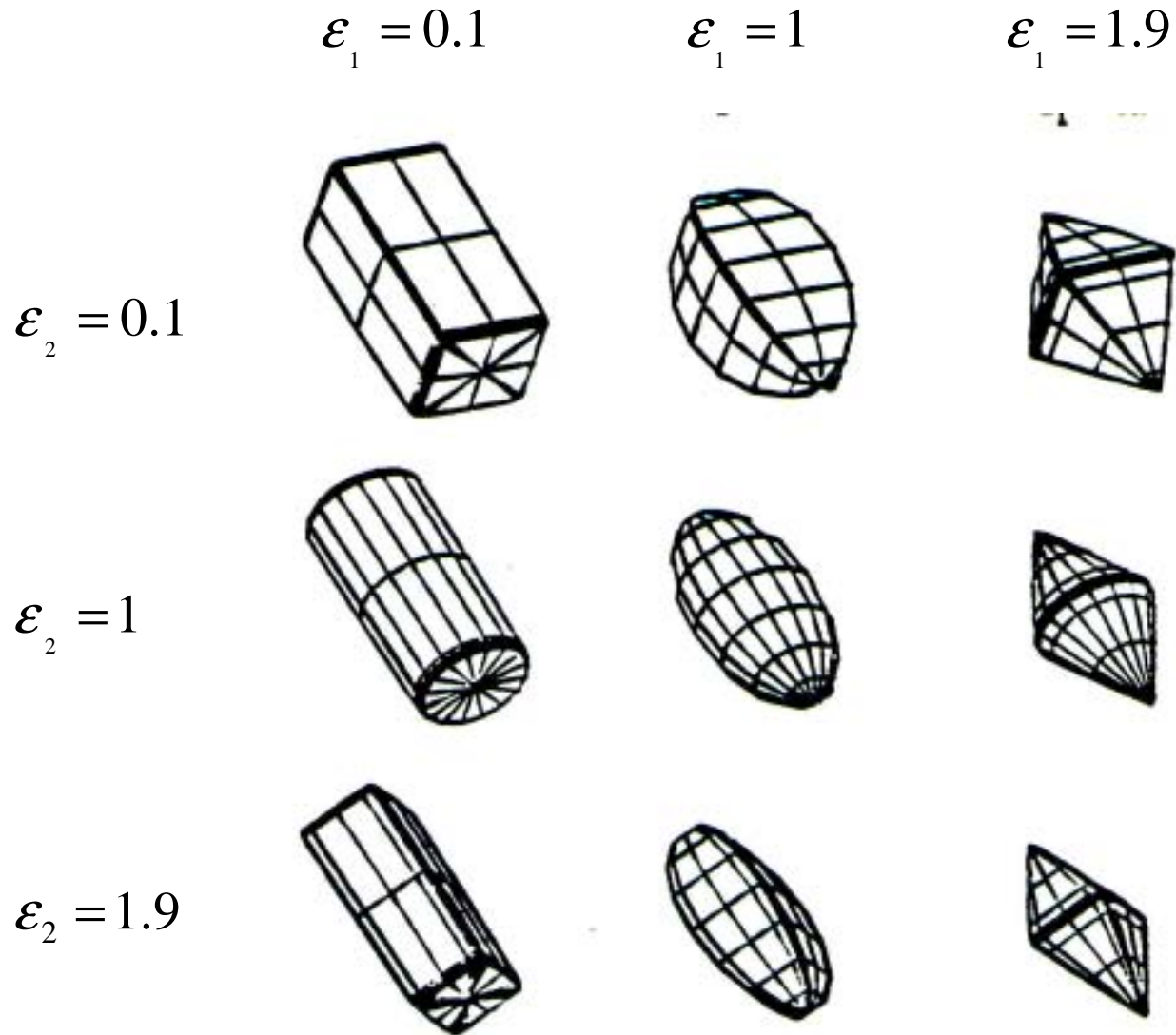
\uparrow $\epsilon_1 = \epsilon_2 = 1$

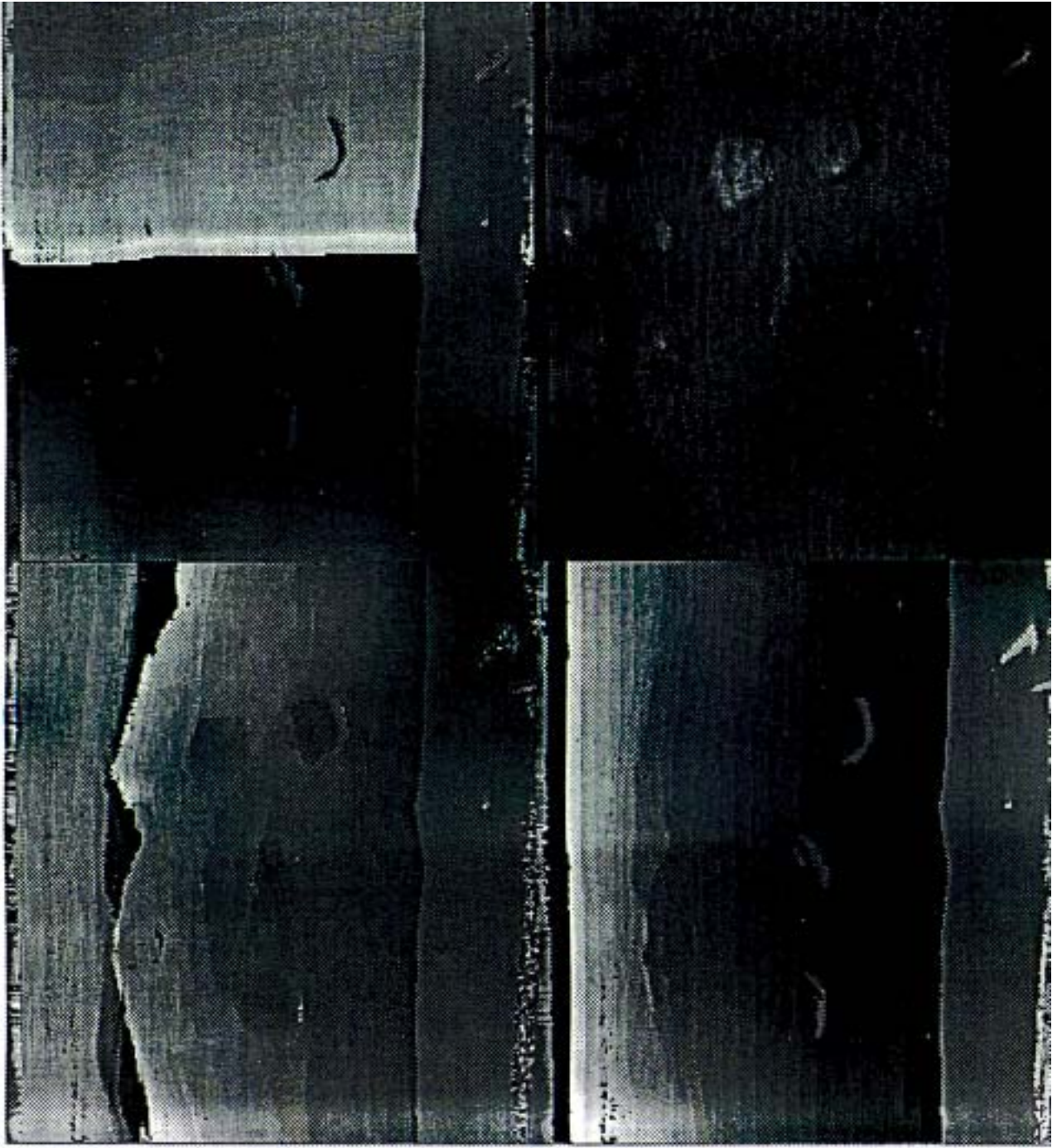
SQ $\left[\left[\left[\frac{x}{a} \right]^{\frac{2}{\epsilon_2}} + \left[\frac{y}{b} \right]^{\frac{2}{\epsilon_2}} \right]^{\frac{\epsilon_2}{\epsilon_1}} + \left[\frac{z}{c} \right]^{\frac{2}{\epsilon_1}} \right]^{\epsilon_1} = 1$

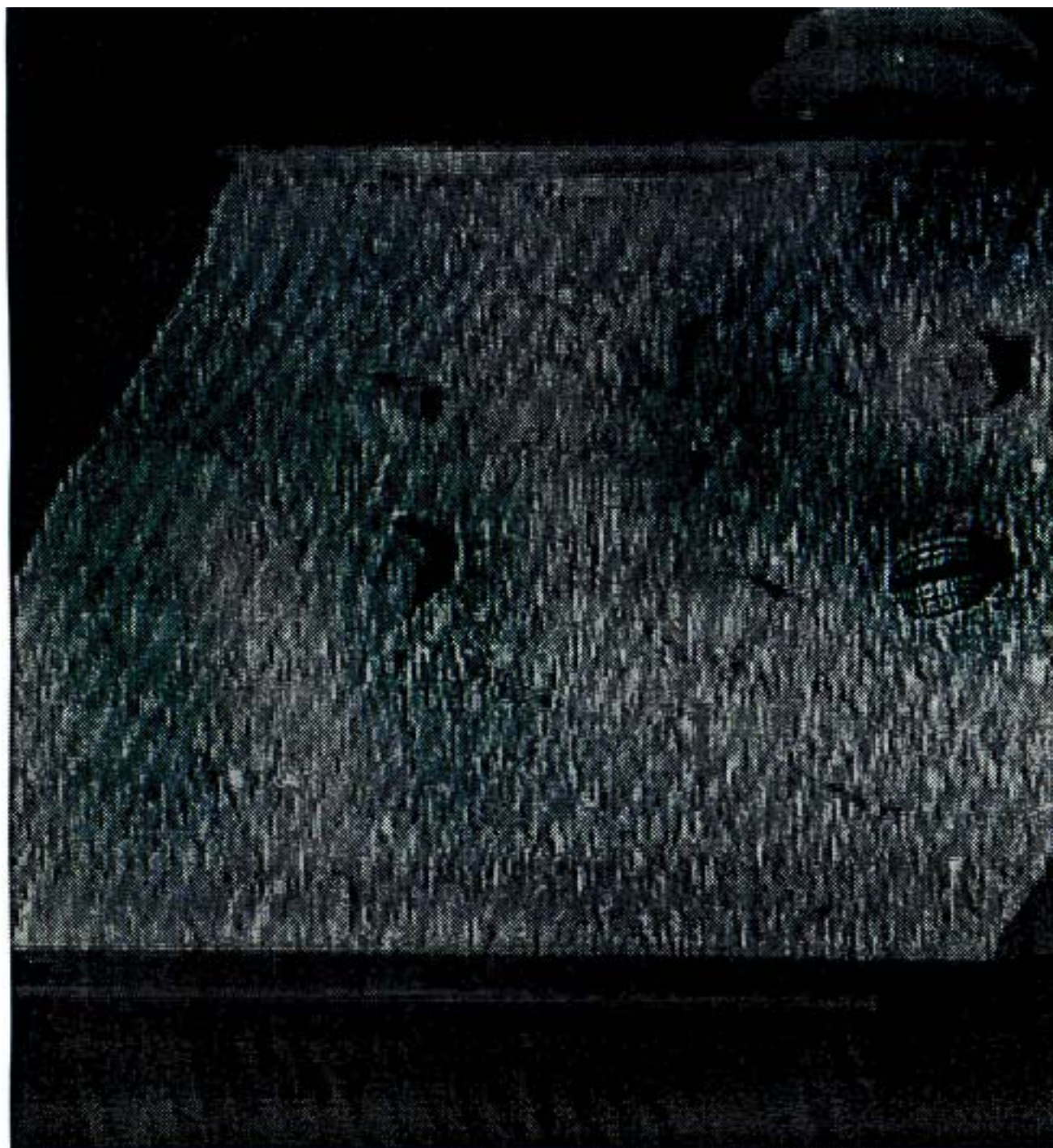
$\left. \begin{array}{l} a \\ b \\ c \end{array} \right\}$ size parameters

$\left. \begin{array}{l} \epsilon_1 \\ \epsilon_2 \end{array} \right\}$ shape parameters

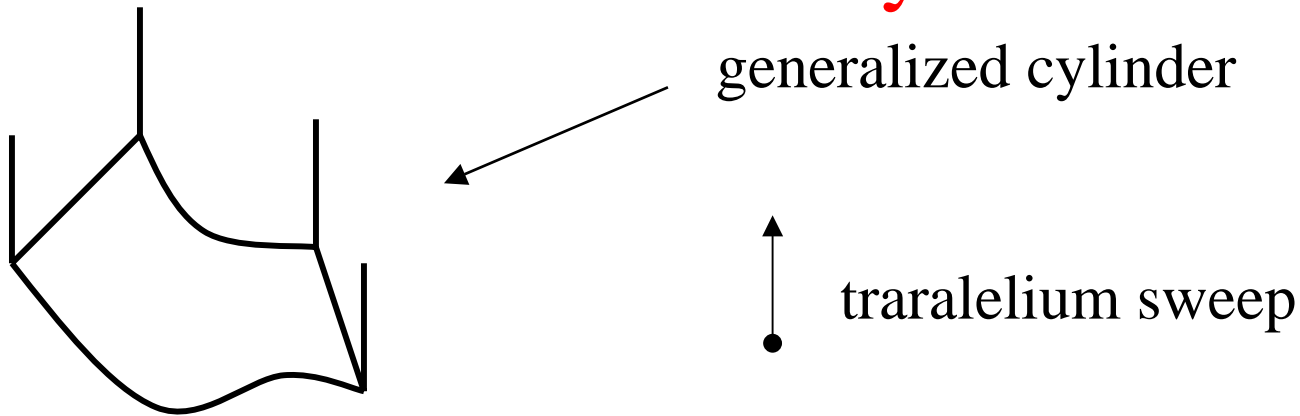
Shape representation using Superquadric



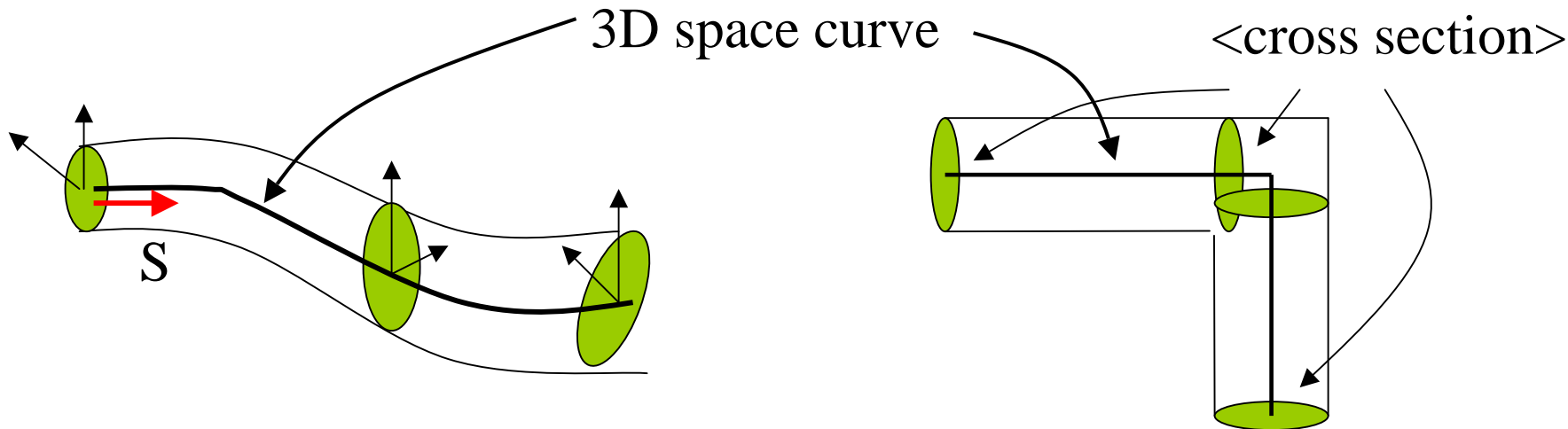




Generalized Cylinder

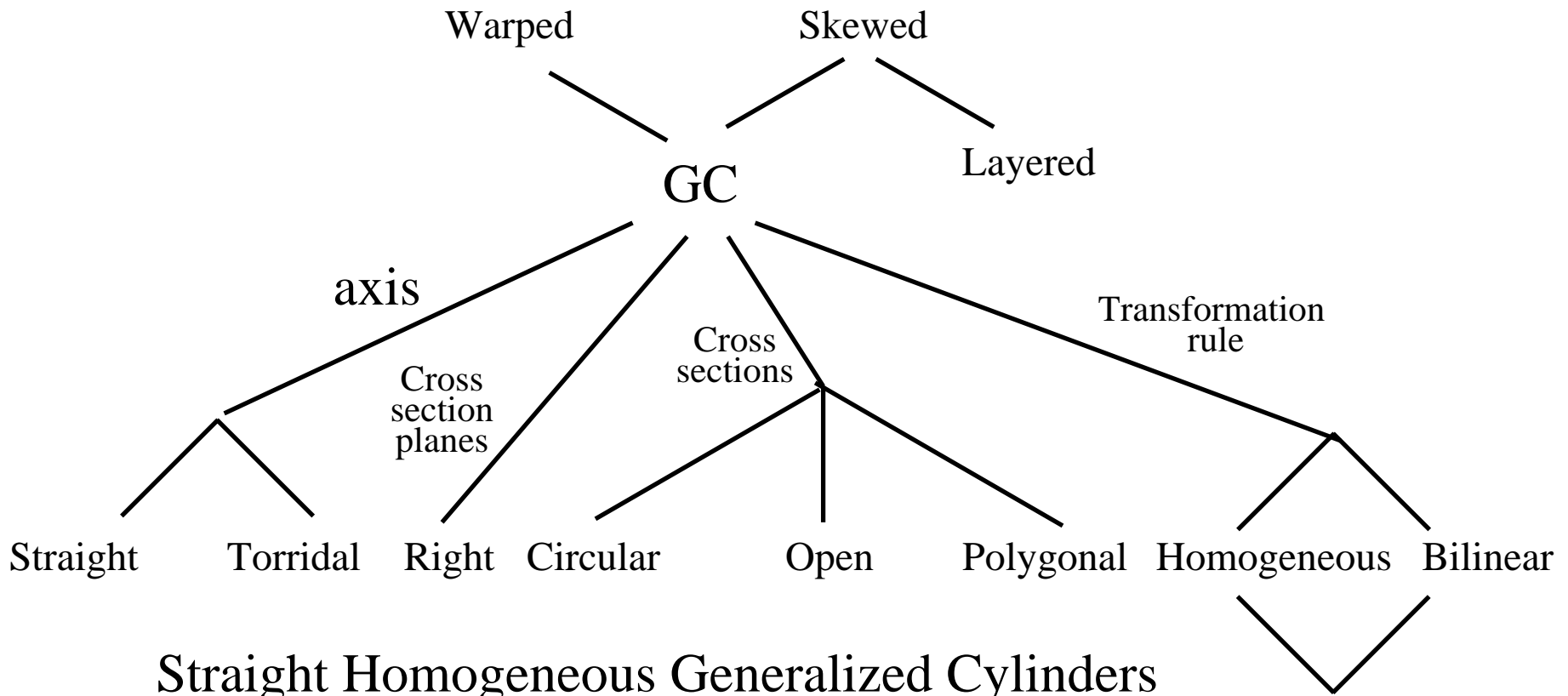


“the sweep volume” of a two-dimensional set moved along a three-space curve

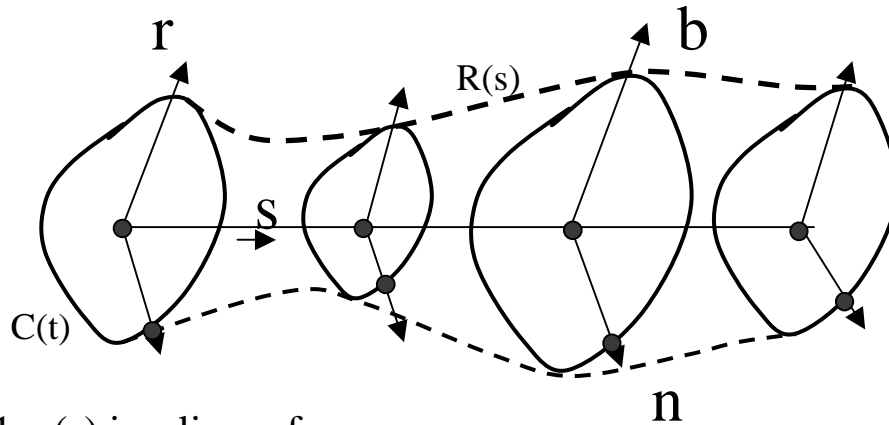


A generalized cylinder and some cross-sectional coordinate systems

A possibly “pathological” situation. Cross sections may be simply described as circles centered on the axis, but then their intersection makes volume calculations (for instance) less straightforward.

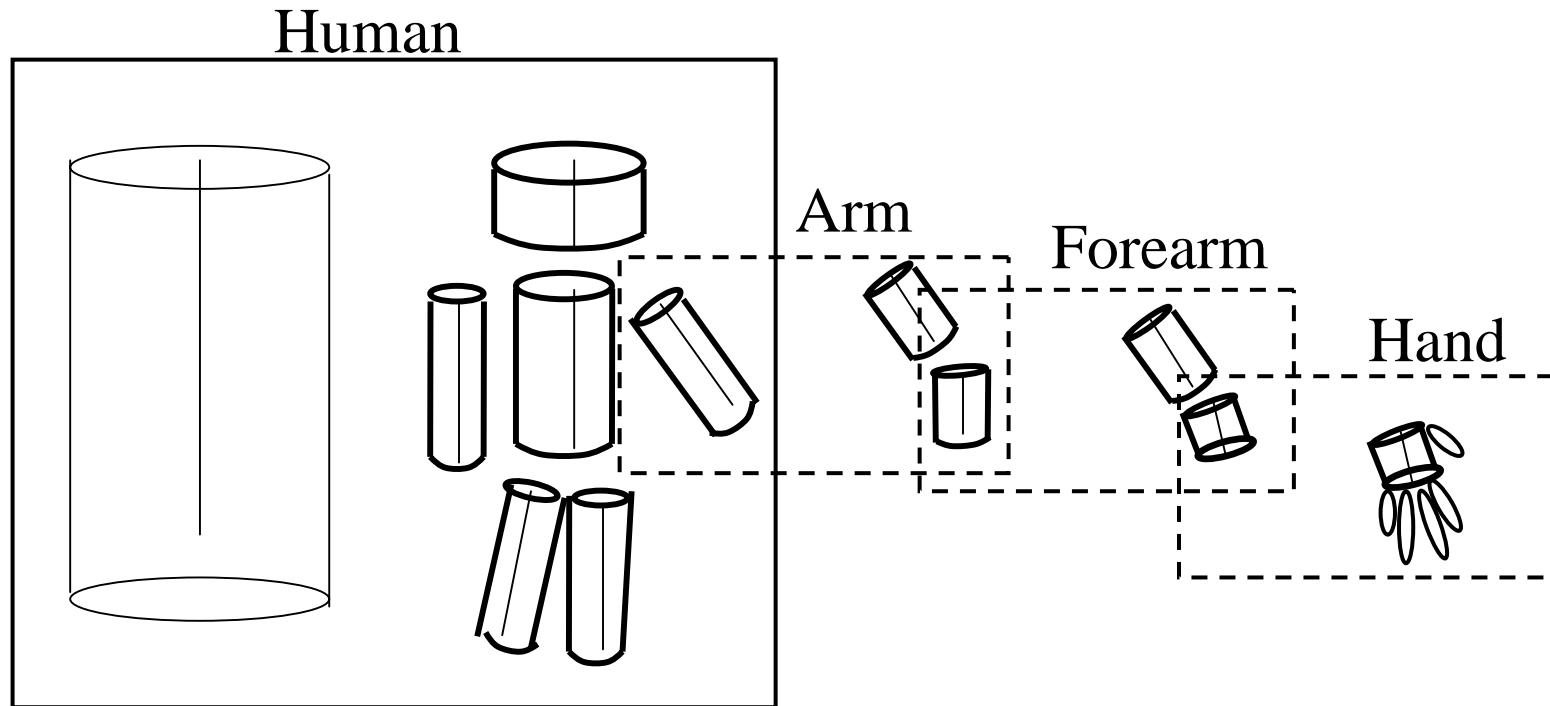


Straight Homogeneous Generalized Cylinders



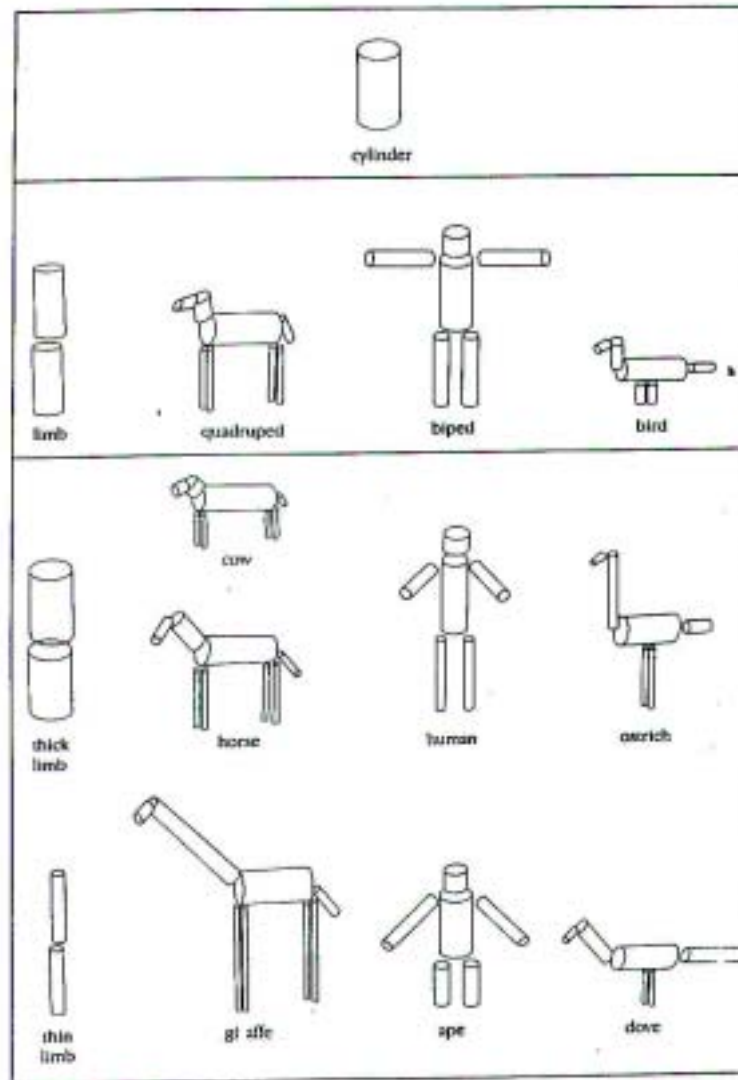
1. $a(s)$ is a linear func
2. n - b planes are parallel
3. All cross sections have the same shape but may vary in size.

Modular representation by GC



This diagram illustrates the organization of shape information in a 3-D model description. Each box corresponds to a 3-D model, with its model axis on the left side of the box and the arrangement of its component axes on the right. In addition, some component axes have 3-D models associated with them, as indicated by the way the boxes overlap. The relative arrangement of each model's component axes, however, is shown improperly, since it should be in an object-centered system rather than the viewer-centered projection used here (a more correct 3-D model is given by the table shown in Figure 5-5c). The important characteristics of this type of organization are: (1) Each 3-D model is a self-centered unit of shape information and has a limited complexity; (2) information appears in shape contexts appropriate for recognition (the disposition of a finger is most stable when specified relative to the hand that contains it); and (3) the representation can be manipulated flexibly. This approach limits the representation's scope, however, since it is only useful for shapes that have well-defined 3-D model decompositions.

Hierarchical Representation by GC



Extracting GC



Initial light-strip image



initial axis estimation



preliminary center and axis estimation



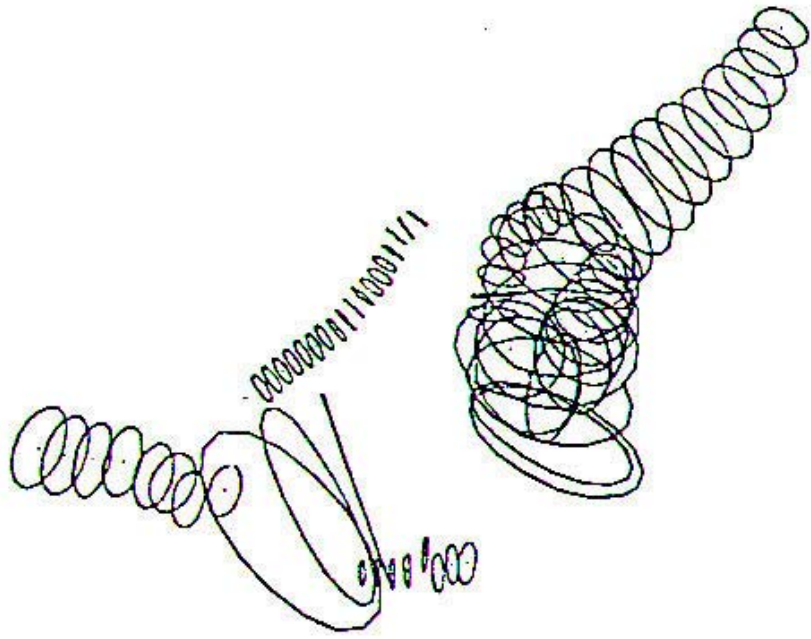
Corn with smoothed radius function



completed analysis

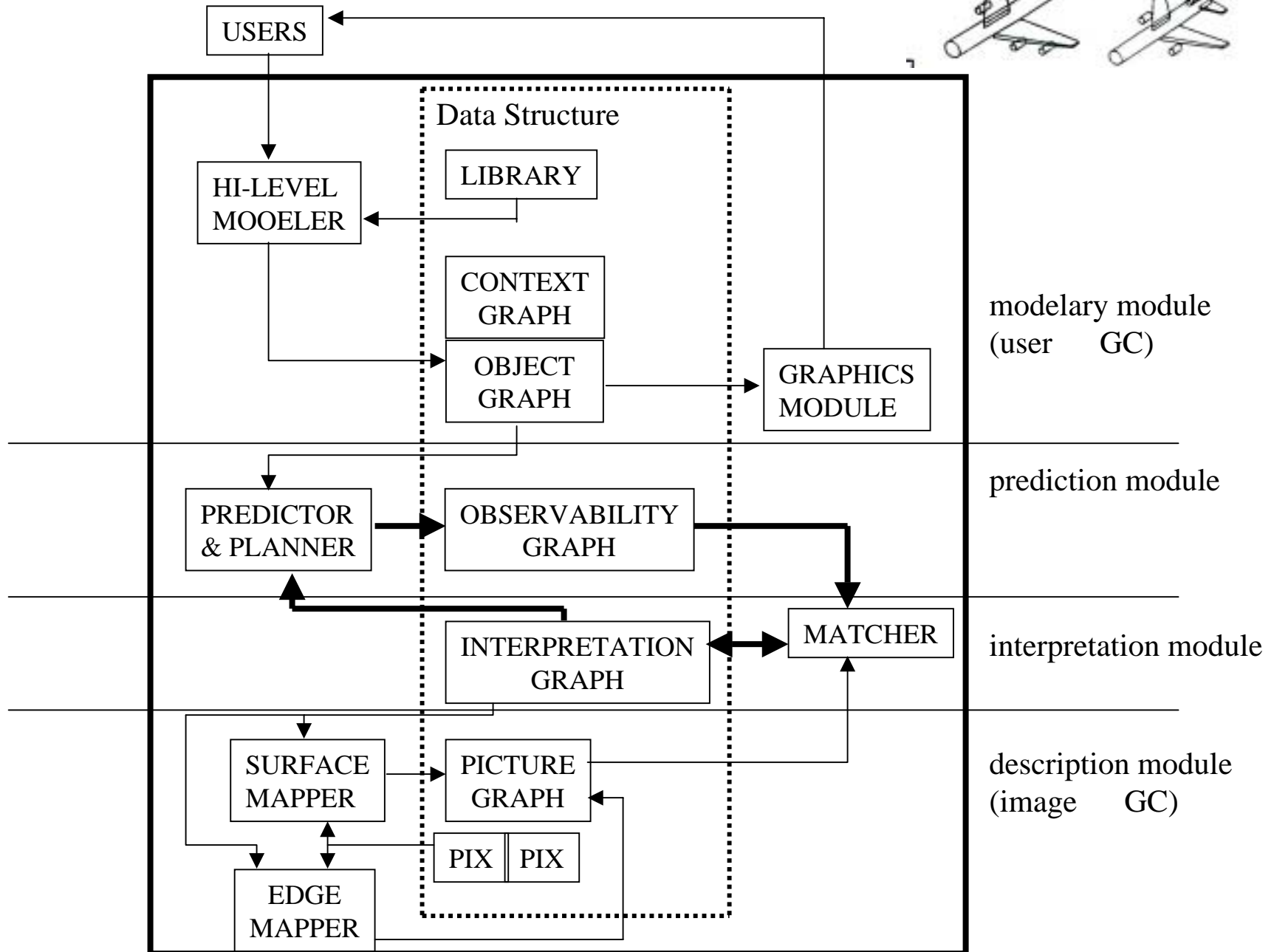


(g)



(d)

ACRONYM system



Back constraints

Prediction of the length of a feature

$$p = (1 \times f) / d$$

$$\rightarrow P = [pl, ph]$$

Now you observe a feature whose possible length is

$$M = [ml, mh]$$

$$N' = M \text{ AND } P = [nl, nh]$$

$$nll < (1 \times f) / d$$

$$nh < (1 \times f) / d$$

new constraints to the parameter space spanned by 1, f and d

Example

$7 < \text{WING-WIDTH} < 12$

$1000 < M < 12000$



$7 < \text{WING-WIDTH} < 10.5677531$

$2199 < M < 3322$



MATCH -- L1011!!!



$2356 < M < 2489$

Interpretation example



Edge image (PIX)

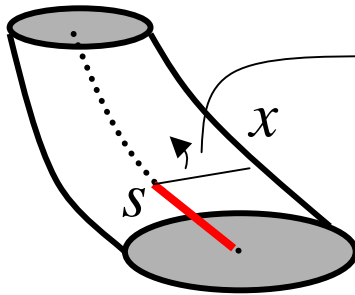


GC description



Interpretation graph

Symmetry seeking representation



$$V(s, x)$$

$r_A(s)$: axis position

$$r(s, x) = V(s, x) - r_A(s)$$

◆ Internal Energy

– energy to maintain the axis position

$$E_{axis} = \int a [\bar{V}(s) - V_A(s)]^2 ds$$

– energy to maintain circular shape

$$E_{cir} = \iint b [\bar{r}(s) - |r(s, x)|]^2 ds dx$$

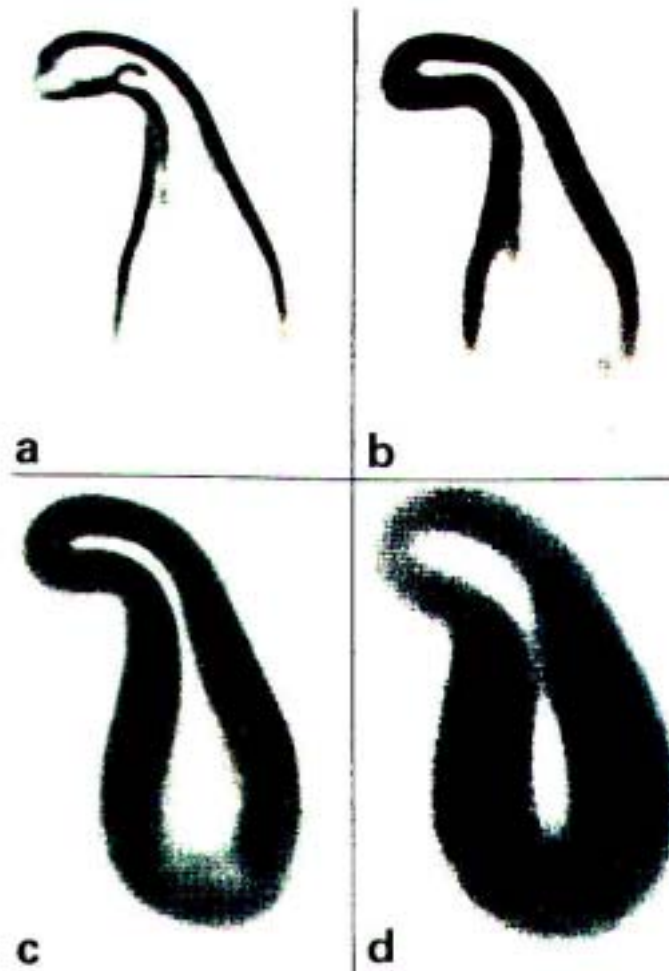
◆ External Energy

– gradient

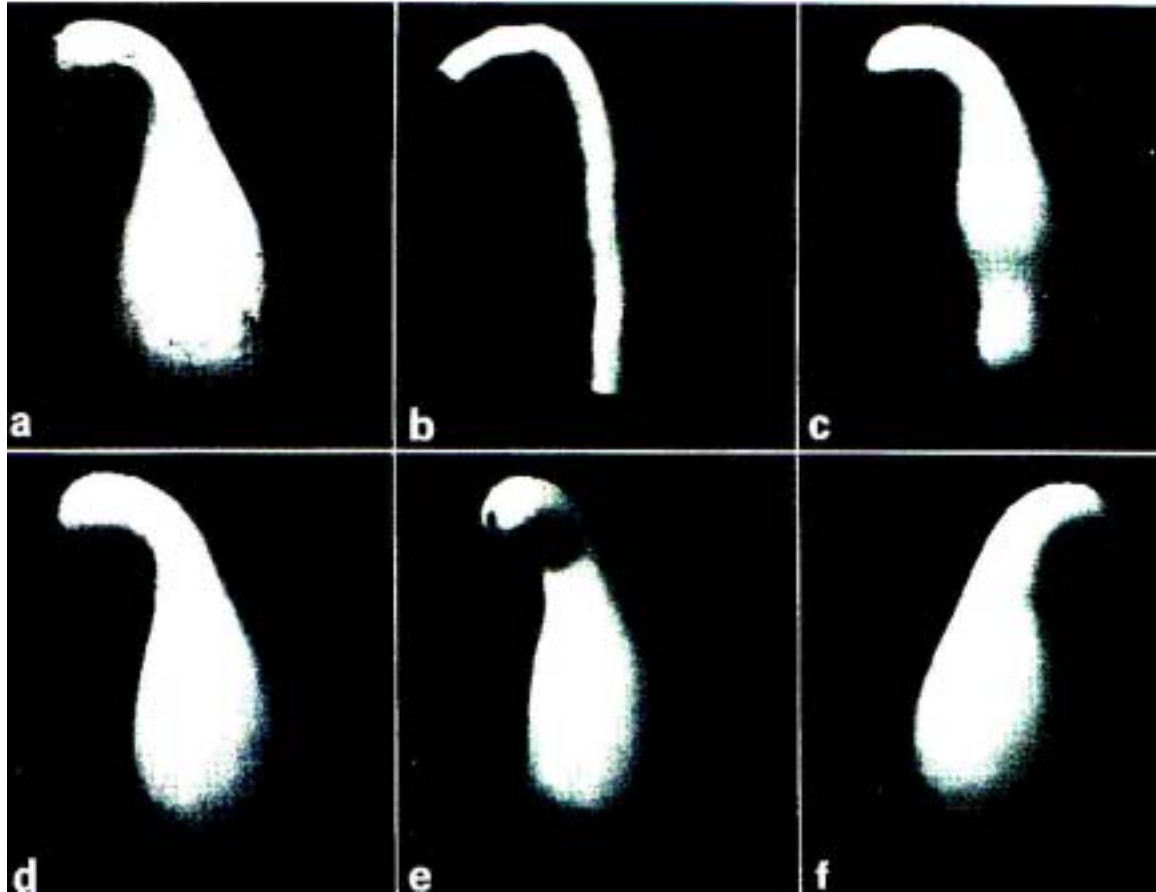
$$E_{grad} = \iint -c [G \cdot \nabla^2 I(V(s, x))]^2 ds dx$$

SSR is a shape such that it minimizes the total energy E

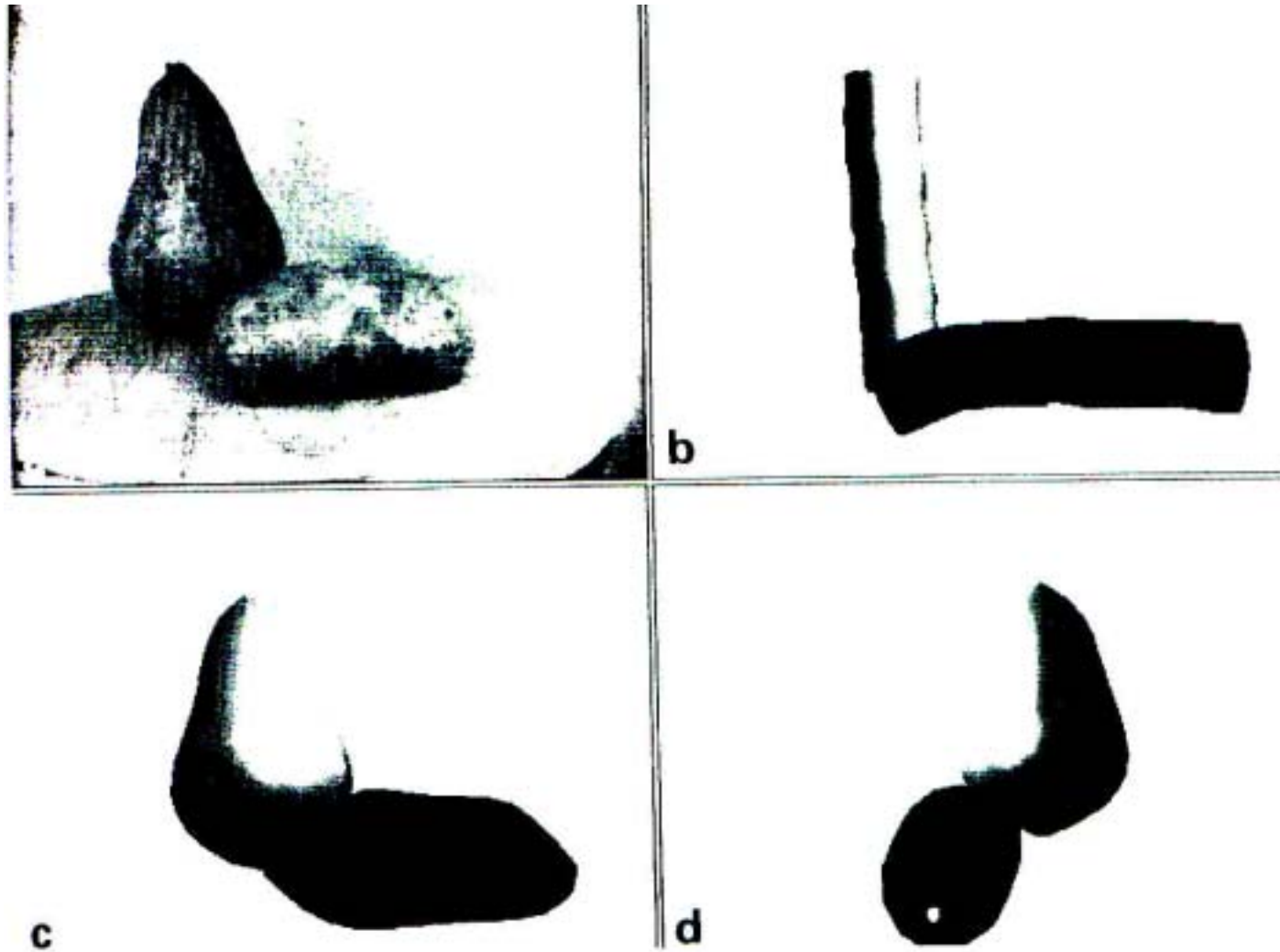
$E = \text{internal energy} + \text{external energy}$



Creating an image potential function. Result of applying image operations to the squash image of Fig.3.(a)-(d) increases progressively. Darkness indicates magnitude of local gradient of the Gaussian blurred image. Each processed image has been resealed to span the available intensity range.

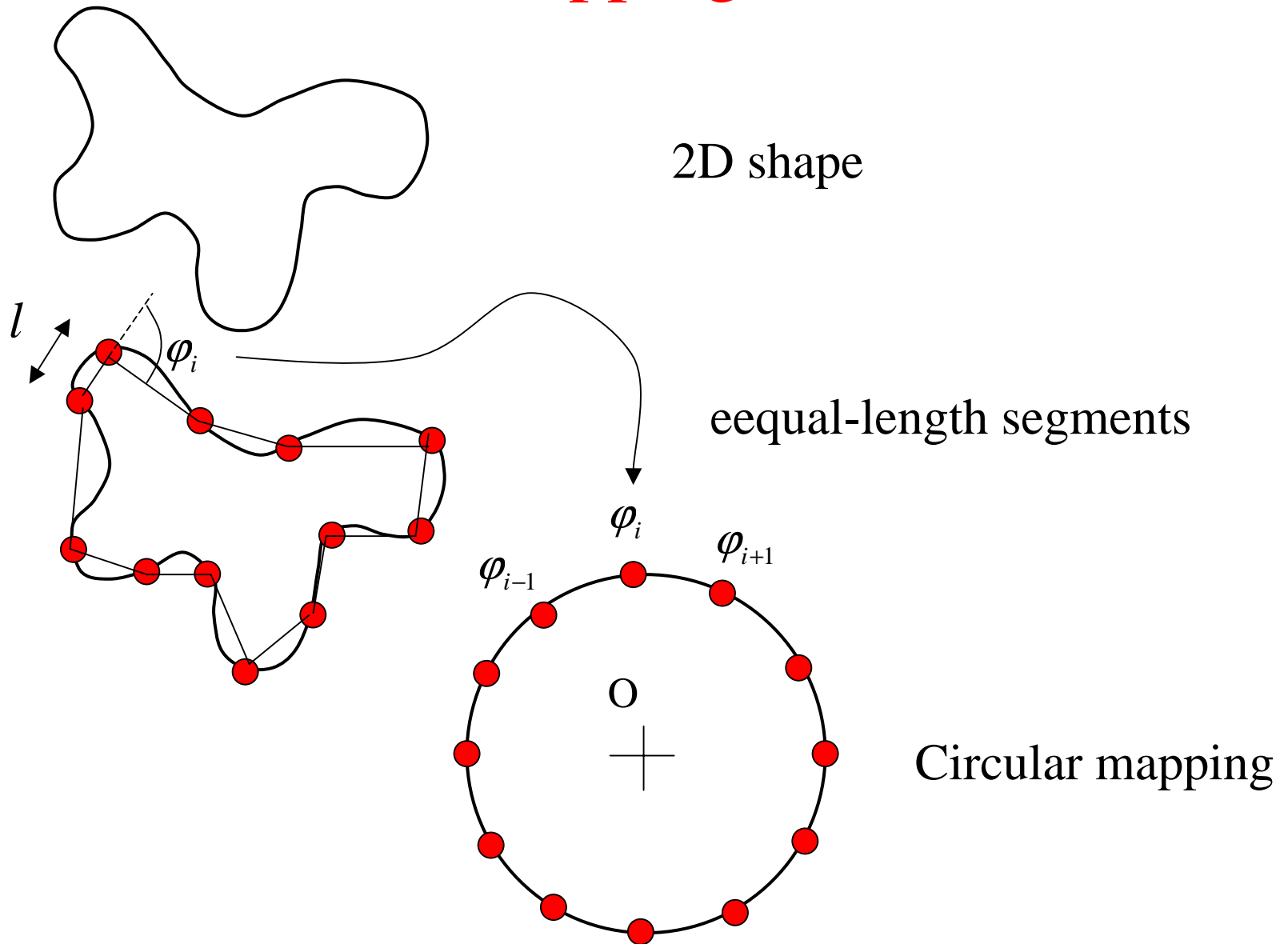


Reconstruction of a squash. (a) Squash image. Selected frames from an animation sequence are shown: (b) Initial state of the 3D model. (c) Intermediate shape during reconstruction. (d) Final reconstructed model. (e)-(f) Model rotating rigidly in space. The 3D model is rendered as a shaded shell.

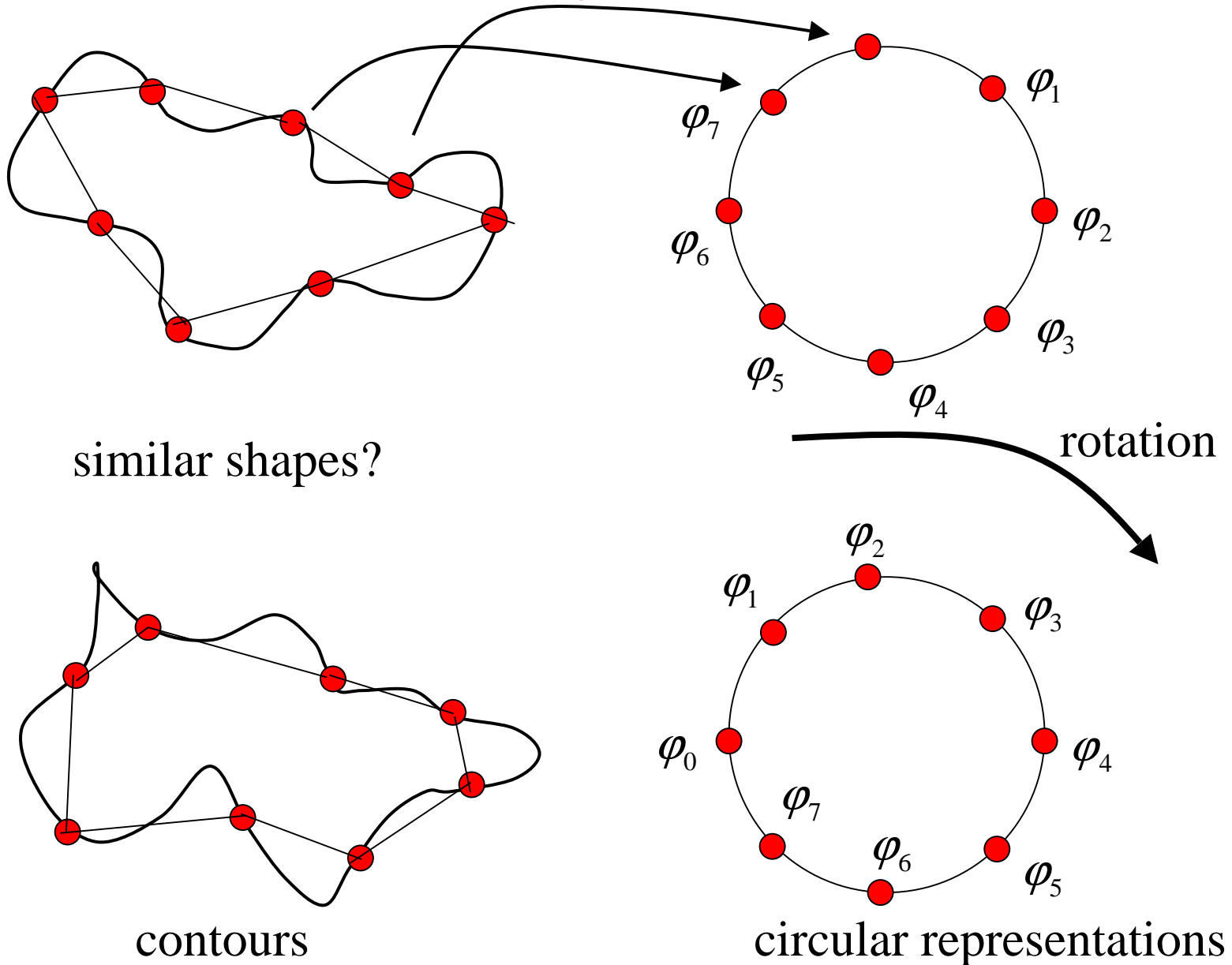


Reconstruction of a still life scene with symmetry-seeking deformable models. (a) Image of the scene. (b) Initial user-specified configurations of the 3D models. Two rendered views of the reconstructed still life: (c) Frontal view of the 3D models; (d) side view.

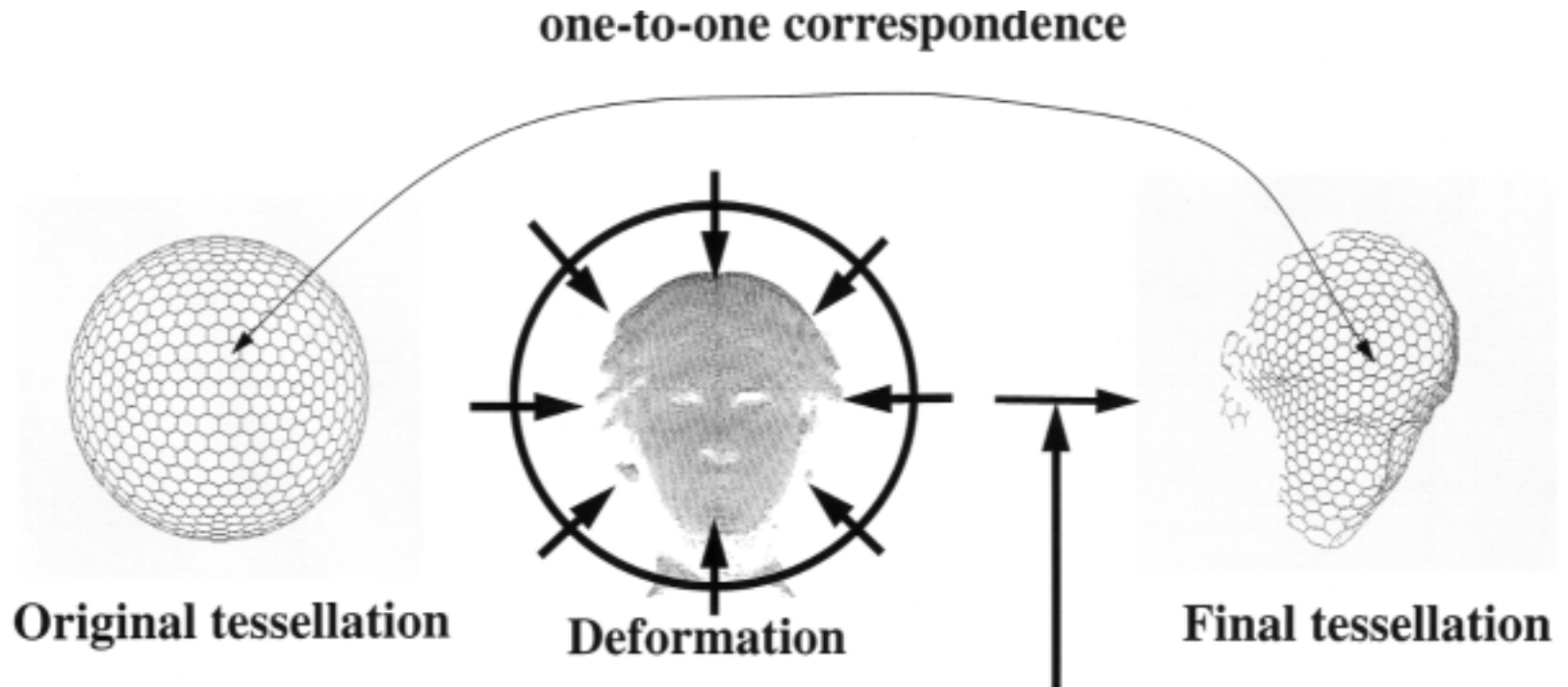
Circular mapping (2D)



Matching (2D)



Deformable Surface Mapping



Regularization

Each Mesh has roughly equal area

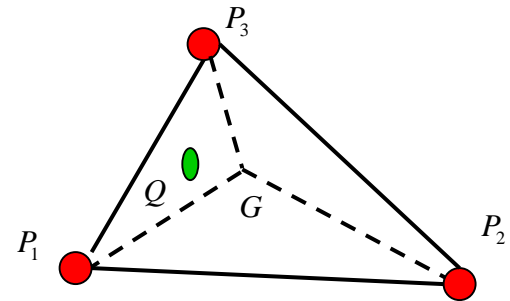
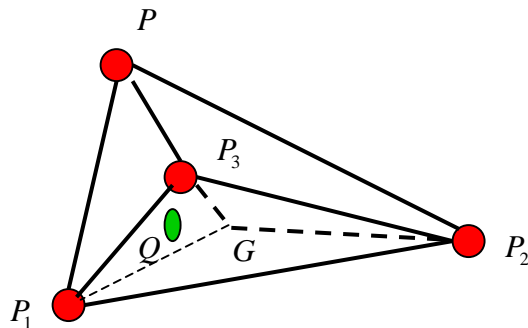
Each mesh has the same topology

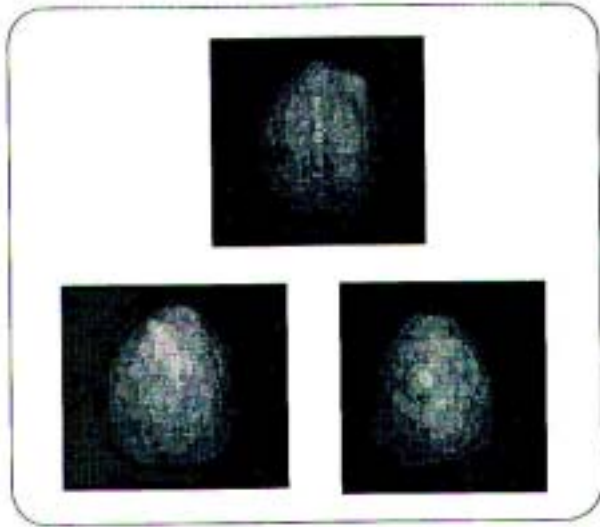
Semi Regular Mesh (3D)

Deform a spherical surface

- internal energy
 - smoothness
- external energy
 - data points
 - features

Regularize the deformed surface
(to have roughly equal area)





Scene



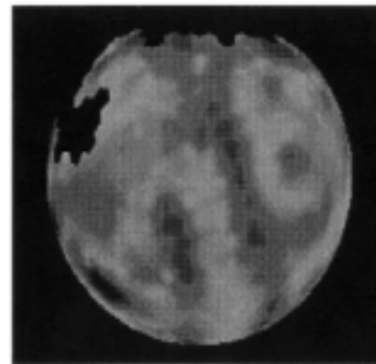
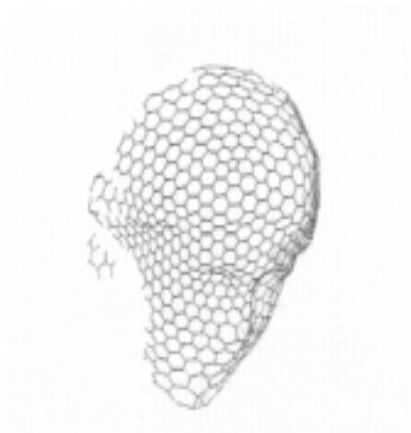
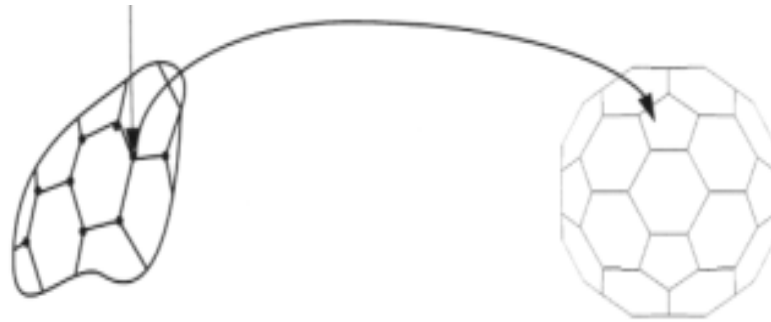
3-D points
from range image



Semi-regular mesh

Spherical Attribute Image (SAI)

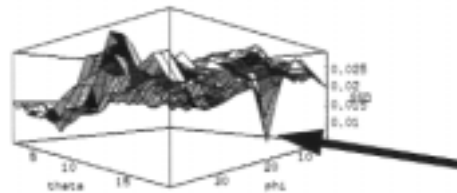
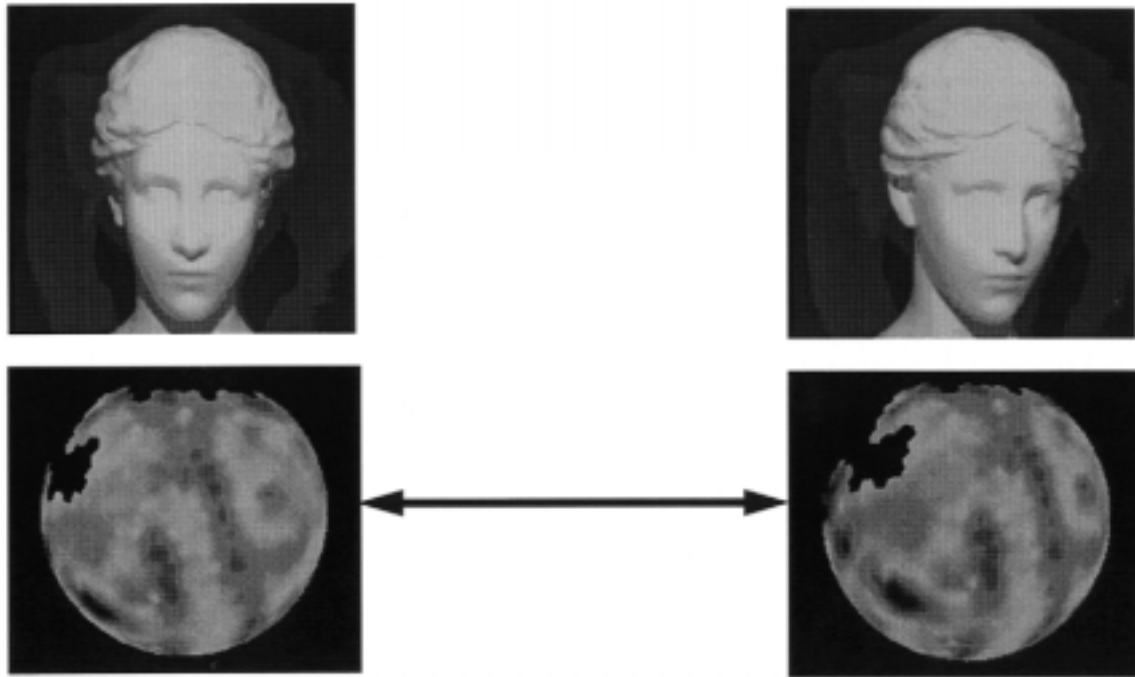
Gaussian Curvature



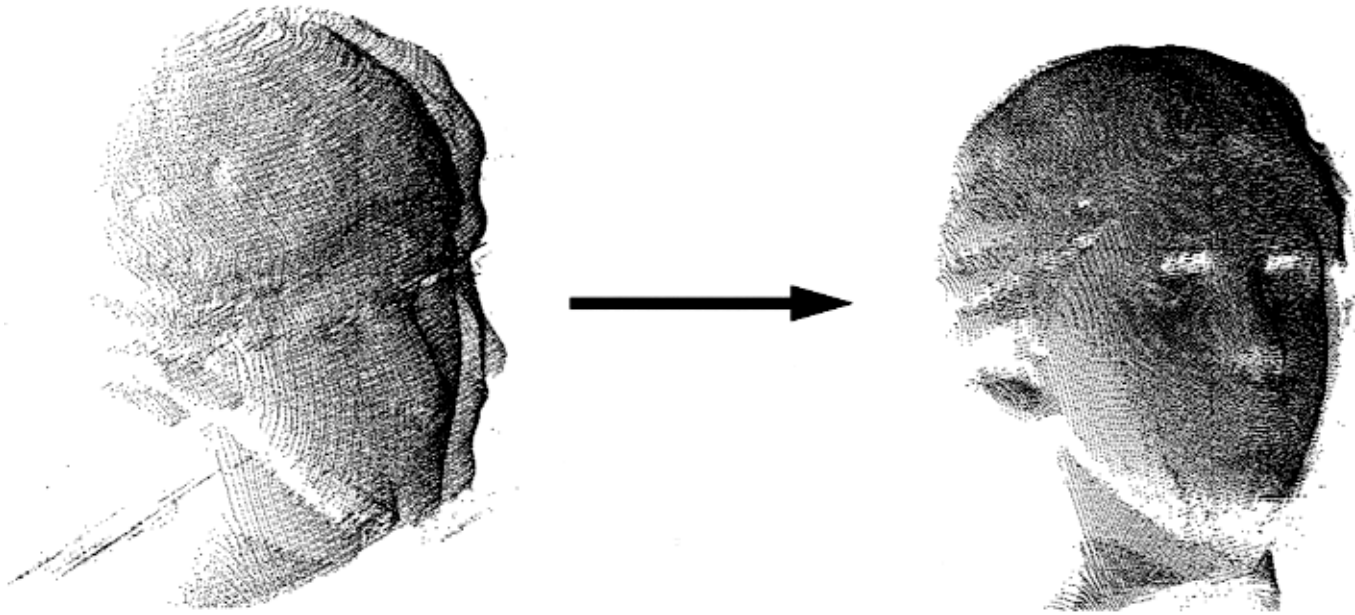
Deformed Surface

SAI

SAI Matching

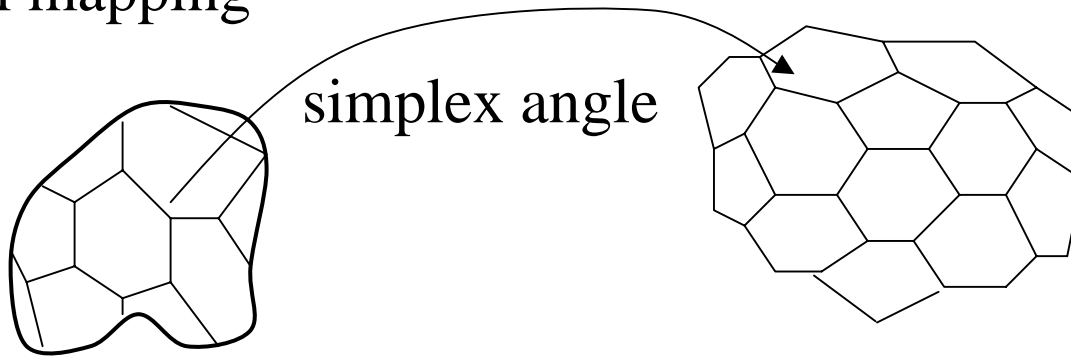


Matching Example

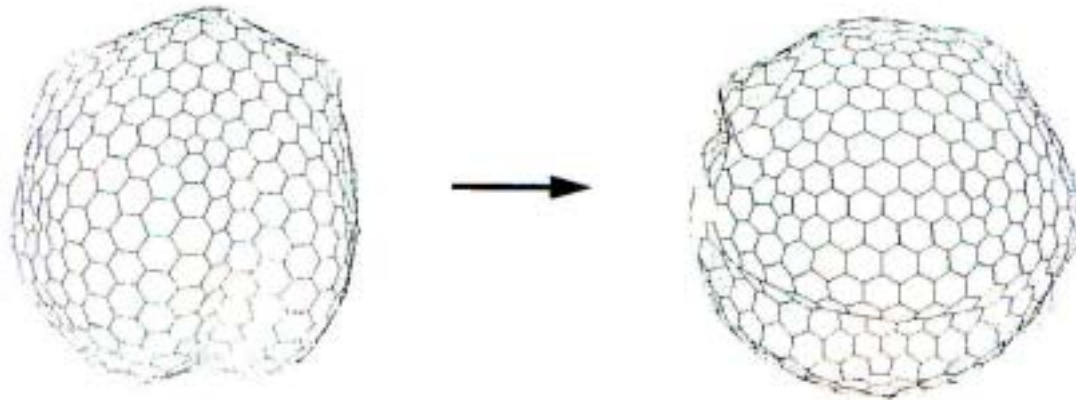


Spherical Attribute Image

Spherical mapping



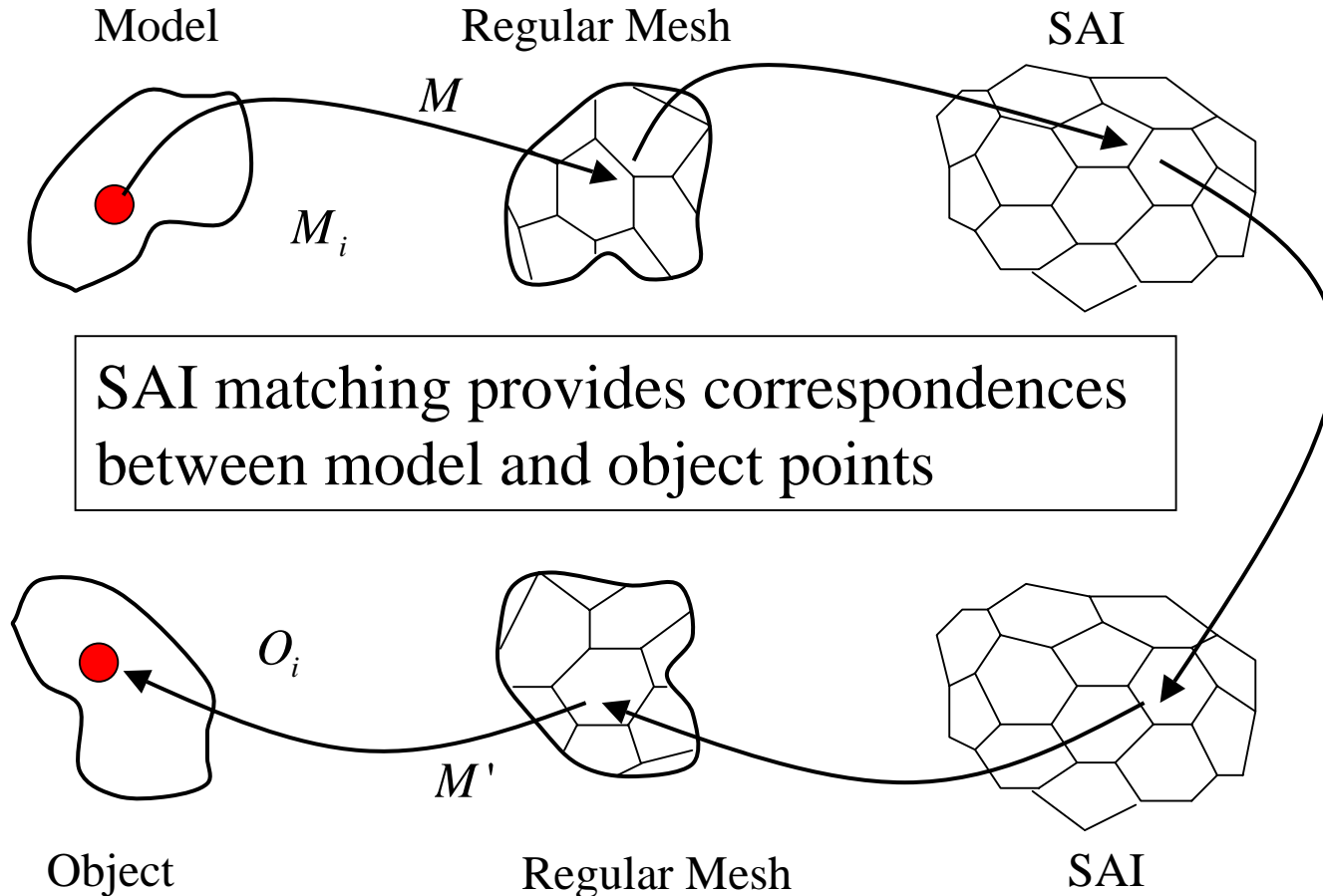
Spherical Attribute Image
using simplex angle



deformed surface

SAI

Determining full transformation

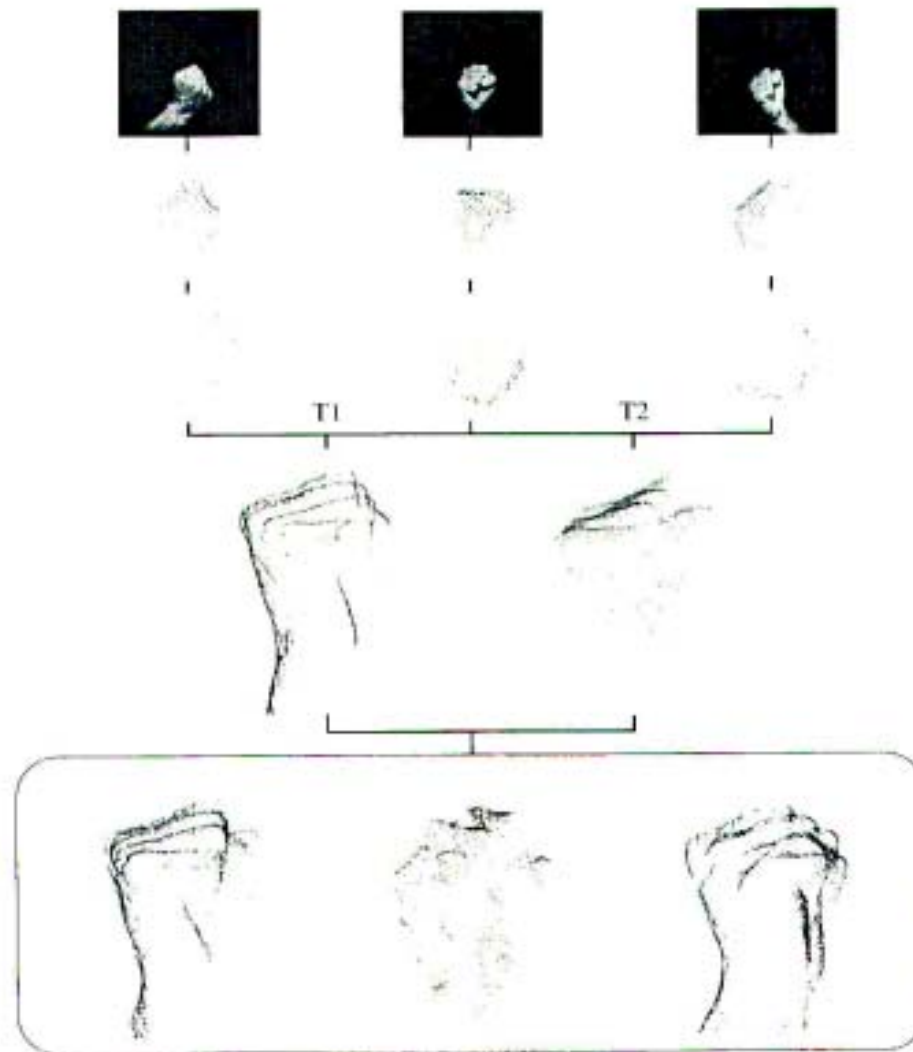


Find R and T to minimize:

$$E = \sum_i |RM_i + T - O_i|^2$$



Merging multiple views (unknown views)



Summary

1. Function based representation
2. Superquadric representation
3. Generalized cylinder representation
4. Symmetry seeking representation
5. Spherical attribute image representation
- 6.

Representing a Solid #2

Computer Vision

9-2

Classification of representation

◆ Surface based representation

- represented as a collection of surfaces
 - » curvature primal sketch -----done
 - » extended gaussian image -----done
 - » aspect graph
 - » b-rep (winged-edge)
 - » well-tessellated surface

◆ Function based representation

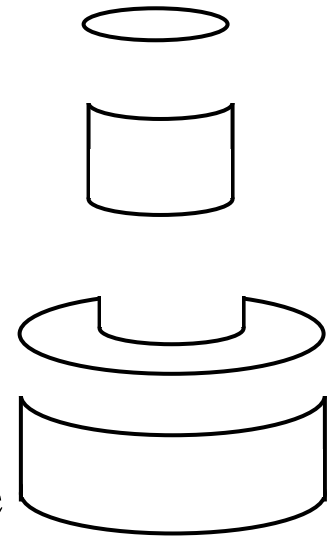
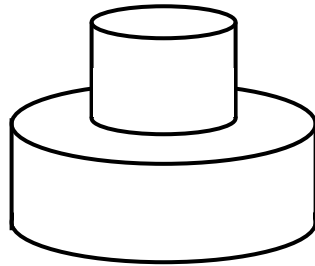
- represented as a function and its parameters
 - » generalized cylinder -----done
 - » superquadric -----done
 - » symmetry seeking -----done
 - » spherical attribute image

◆ Volumetric based representation

- represented as a collection of primitive solids
 - » constructive solid geometry
 - » occupancy graph
 - » oct-tree

Surface based representation

- Represent a solid using a collection of surfaces



◆ which surfaces

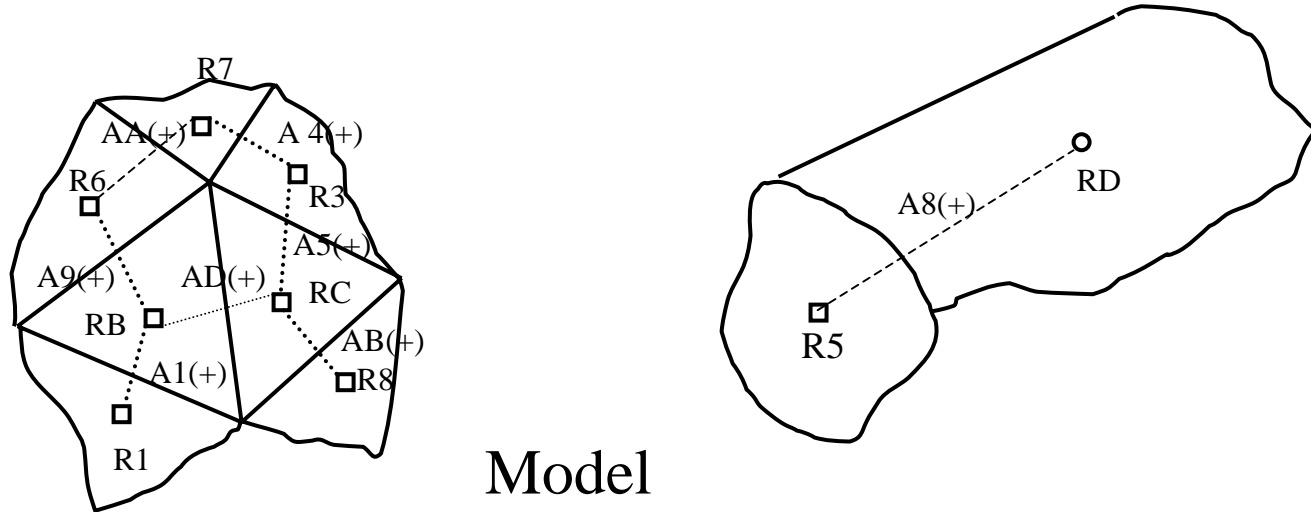
- * collection of visible surfaces -- easy to use
- * collection of all surfaces -- easy to maintain

◆ what is a reasonable definition

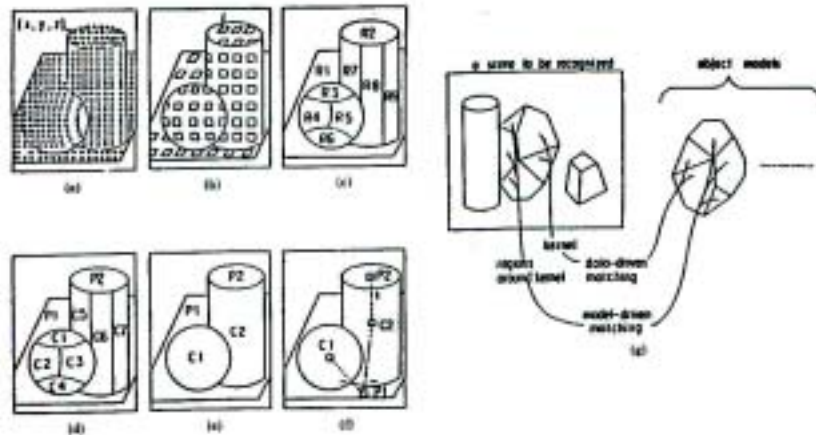
- * orientation discontinuity -- if range data is available, one of the most common definition
- * Gaussian curvature and mean curvature Represent a solid using a collection of surfaces
- * color (brightness) -- view dependent only

Oshima-shirai system

- ◆ View dependent surface based representation
- ◆ orientation discontinuity



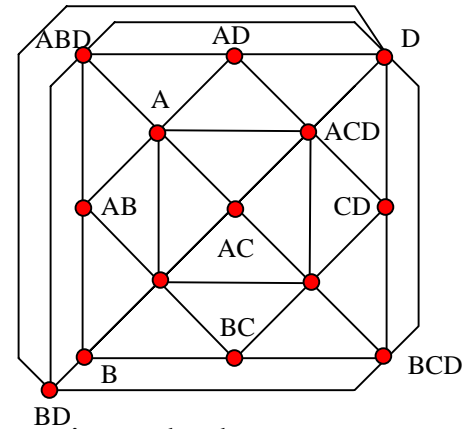
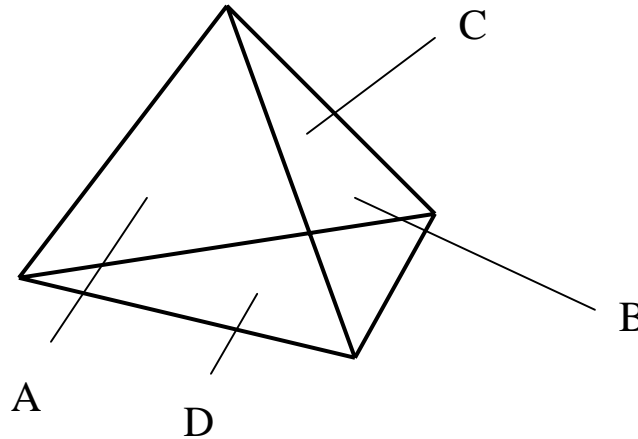
Model



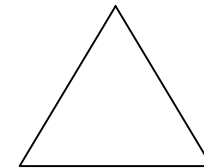
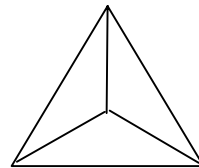
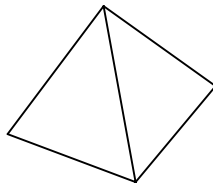
System

Aspect representation

- ◆ How many different representations are necessary?
- ◆ Aspect -- topologically equivalent class of appearances



The aspect graph for a tetrahedron. Nodes are of three types representing whether one, two, or three faces are visible. Arcs connect a pair of nodes when some arbitrarily small change in viewing direction suffices to change the set of visible faces from the faces visible at one node to those visible at the other node without going through any intermediate set of visible faces.

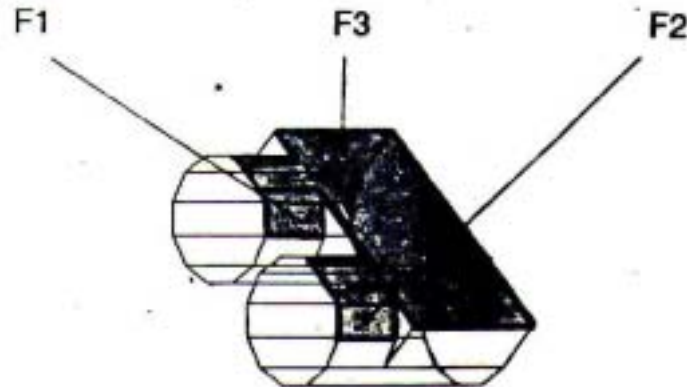
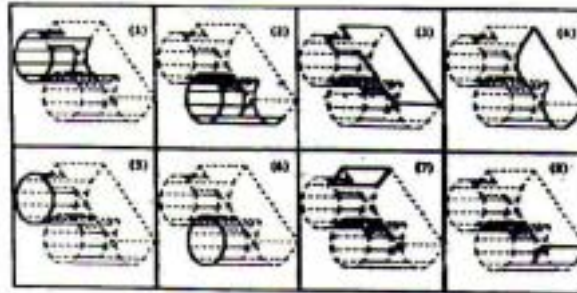


Extracting possible aspects

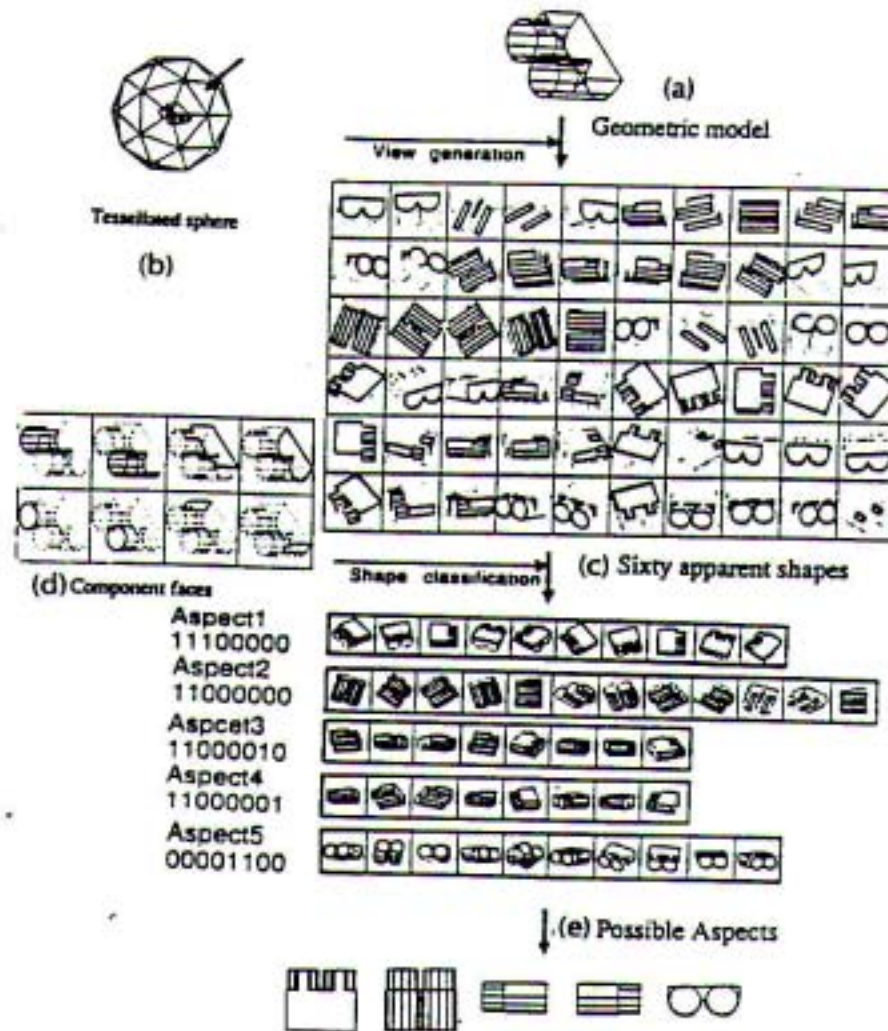
Face Label

F1	F2	F3	F4	F5	F6	F7	F8
1	1	1	0	0	0	0	0

Visible Invisible



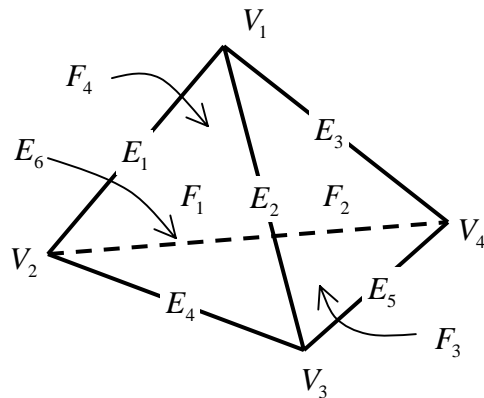
Aspect generation



View independent surface based model

◆ Graph based representation

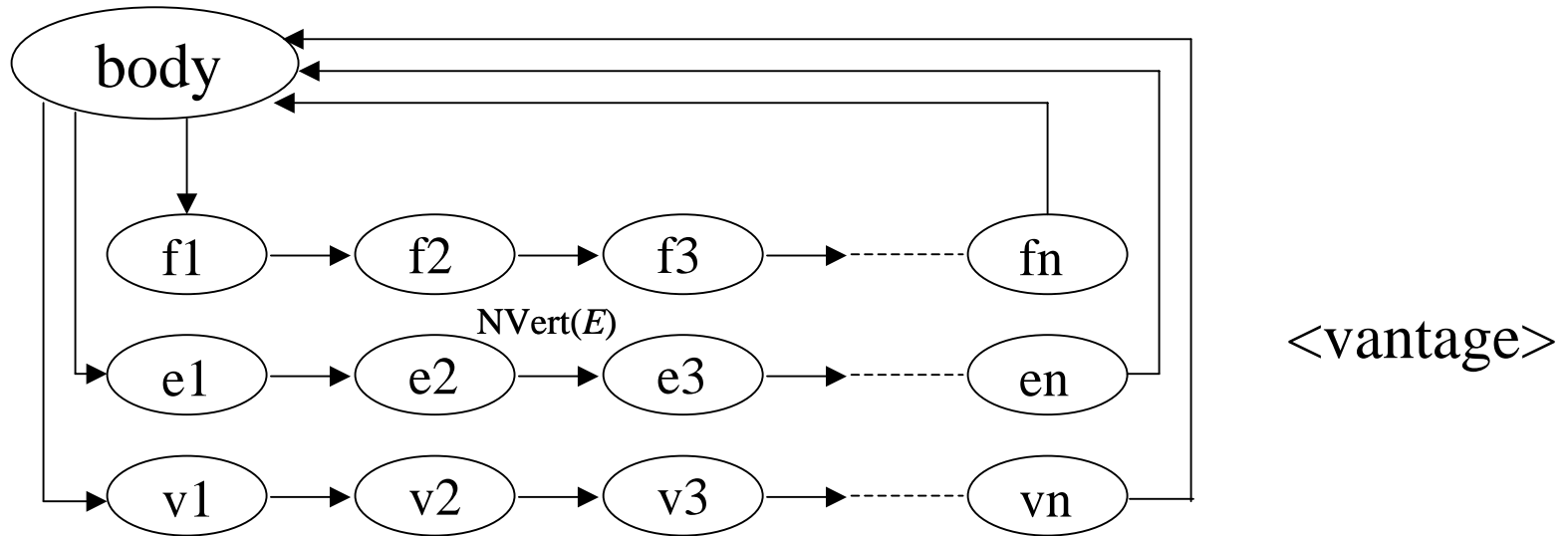
- as a list of the object's faces, edges, vertices
 - » topology or connectivity between vertices, edges and faces
 - » numerical data defining surface equations and vertex coordinates



Vertices	Edges	Faces
V_1	E_1	F_1
$V_2 \ V_3 \ V_4$ $E_1 \ E_2 \ E_3$ $F_1 \ F_2 \ F_4$	$V_1 \ V_2$ $E_2 \ E_3 \ E_4 \ E_6$ $F_1 \ F_4$	$V_1 \ V_2 \ V_3$ $E_1 \ E_4 \ E_2$ $F_2 \ F_3 \ F_4$
V_2	E_2	F_2
$V_1 \ V_3 \ V_4$ $E_1 \ E_4 \ E_6$ $F_1 \ F_3 \ F_4$	$V_1 \ V_3$ $E_1 \ E_3 \ E_4 \ E_5$ $F_1 \ F_2$	$V_1 \ V_3 \ V_4$ $E_2 \ E_5 \ E_3$ $F_1 \ F_3 \ F_4$
V_3	E_3	F_3
$V_1 \ V_2 \ V_4$ $E_2 \ \dots$	$V_1 \ V_4$ $E_1 \ E_2 \ \dots$	$V_4 \ V_3 \ V_2$ $E_4 \ \dots$

Winged edge representation

- Represent topological relationship in a compact manner



- VANTAGE

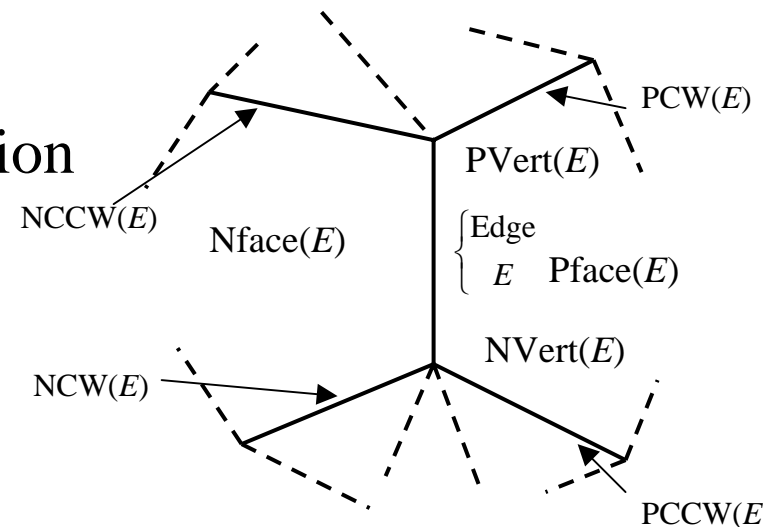
topological information

*edge --- Winged edge representation

geometric information

*face --- face equation

*vertex --- vertex coordinates



Surface based representation on sphere (Well-tessellated sphere rep)

- ◆ represent directions as points on the unit sphere
(usually geodesic dome)

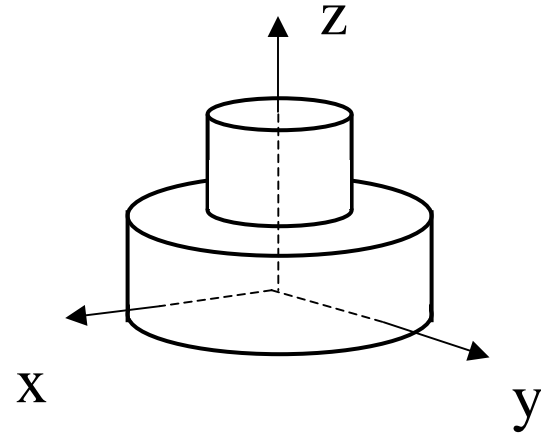
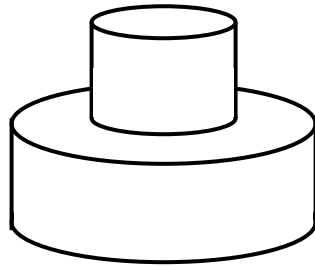


- ◆ moving the points on the sphere out by their associated magnitude (usually the distance between a surface point and the mass center)



Volumetric representation

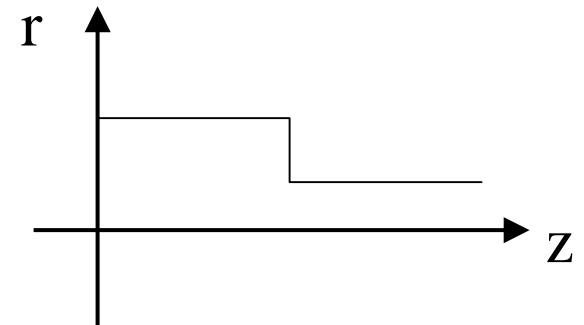
represent a 3D object by its own coordinate system



$$r(z) \geq x^2 + y^2$$

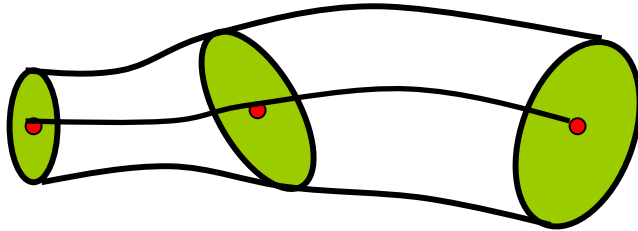
1. easy to store

2. difficult to handle



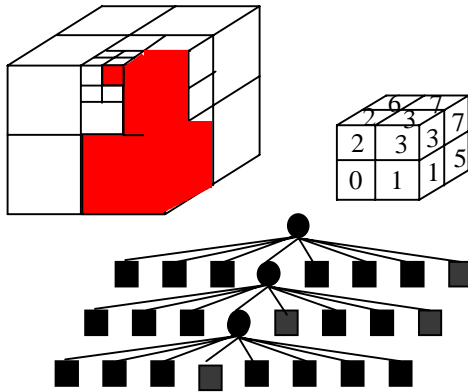
Example of Volumetric representation

a. generalized cylinder



“the sweep volume” of a two D shape moved along a 3D curve

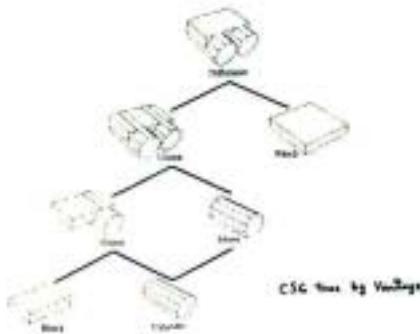
b. oct-tree



“spatial occupancy”

- Node with descendants
- Node representing an empty region
- Node representing a full region

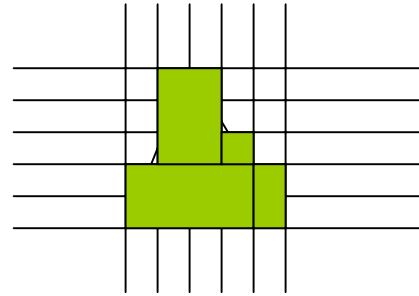
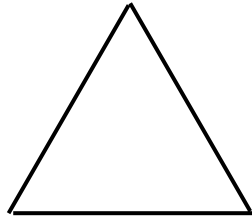
c. constructive solid geometry



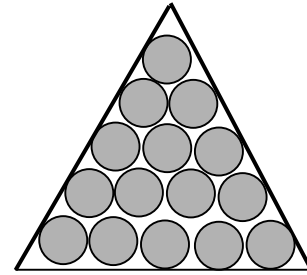
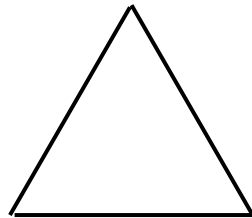
“primitive objects and set operations between them”

Volume based representation

- ◆ Represent a solid as a collection of primitive solids
 - spatial occupancy enumeration



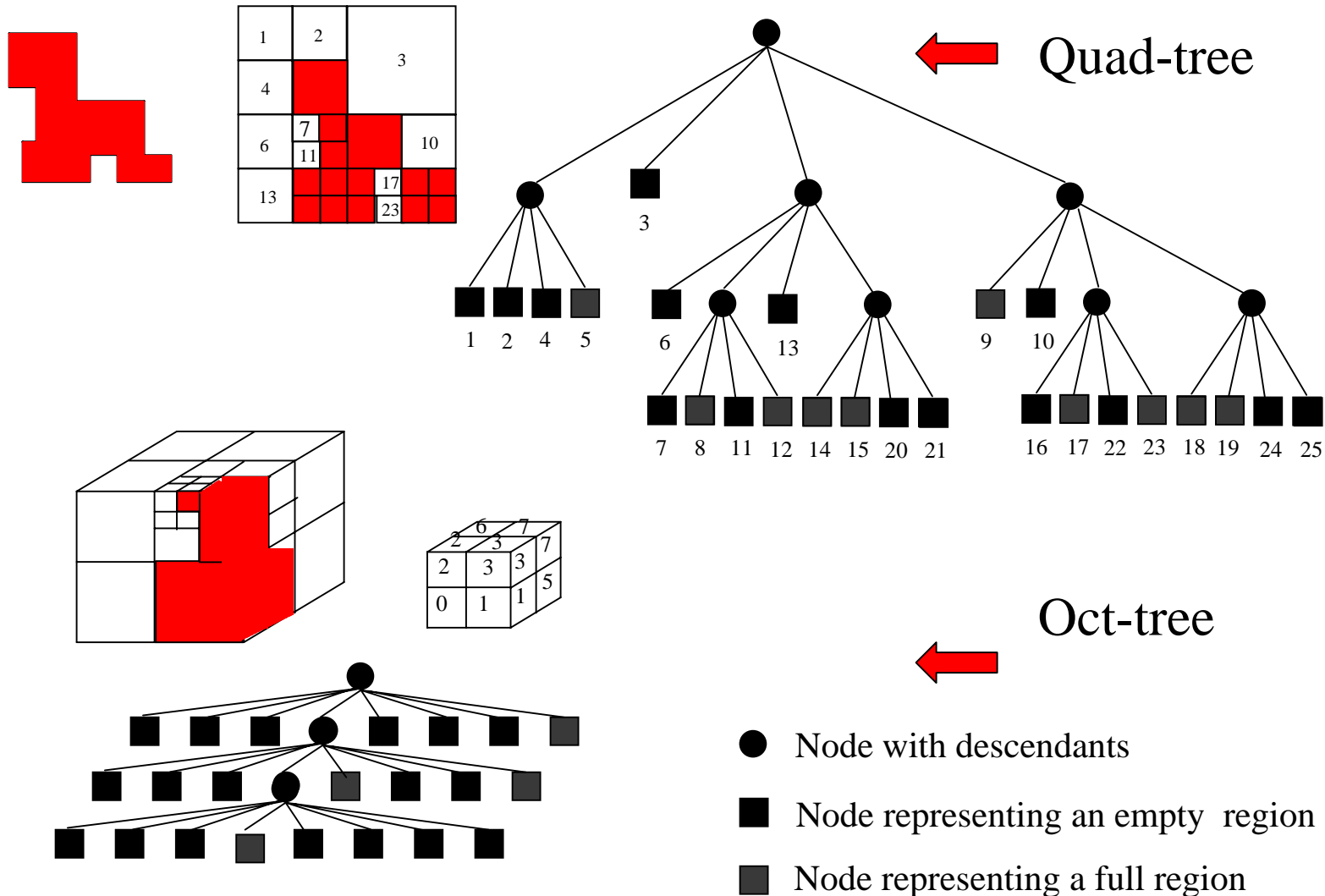
- ◆ overlapping sphere



- ◆ requires many spheres or voxels to represent to represent a relatively simple smooth solid

Oct tree representation

Oct tree encoding -- extension into three dimensions of quad tree encoding



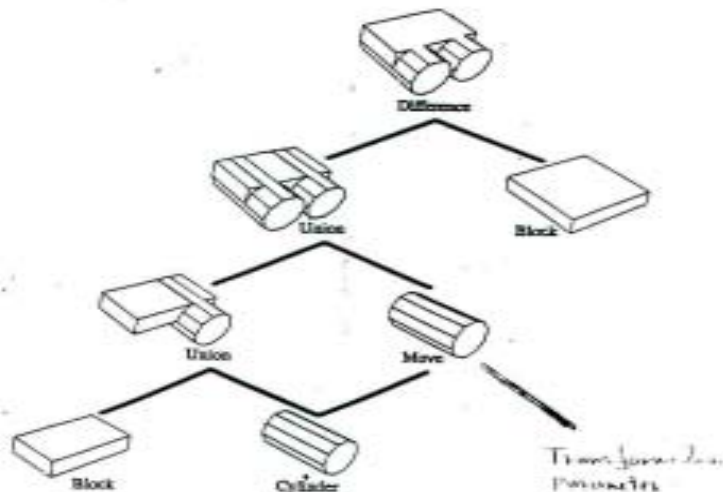
Constructive solid geometry representation

$\langle \text{CSG rep} \rangle :: \langle \text{primitive solid} \rangle |$
 $\text{move } \langle \text{CSG Rep} \rangle \text{ by } \langle \text{Motion params} \rangle$
 $\langle \text{CSG Rep} \rangle \langle \text{operation} \rangle \langle \text{CSG Rep} \rangle$

leaf node: either primitive or motion params

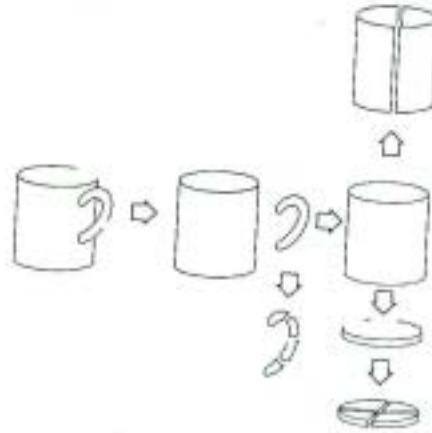
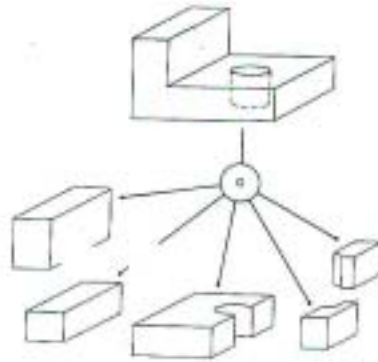
other node: Boolean operations (UNION, DIF, INT)

- ◆ Input to a CAD system
- ◆ boundary-evaluation function converts CSG to boundary representation such as those based on winged-edge representation



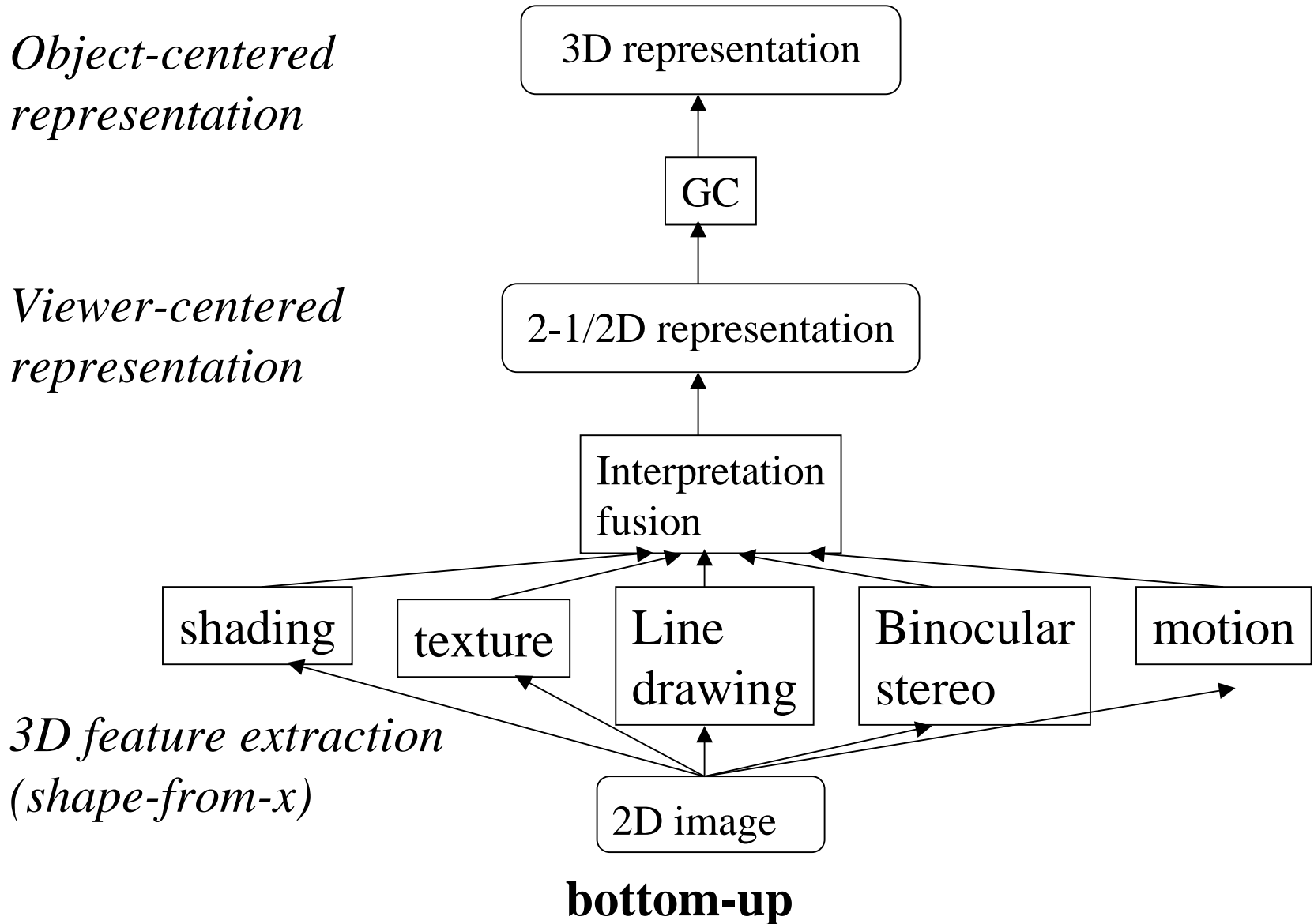
Cell decomposition representation

- quasi-disjoint (do not share volume)

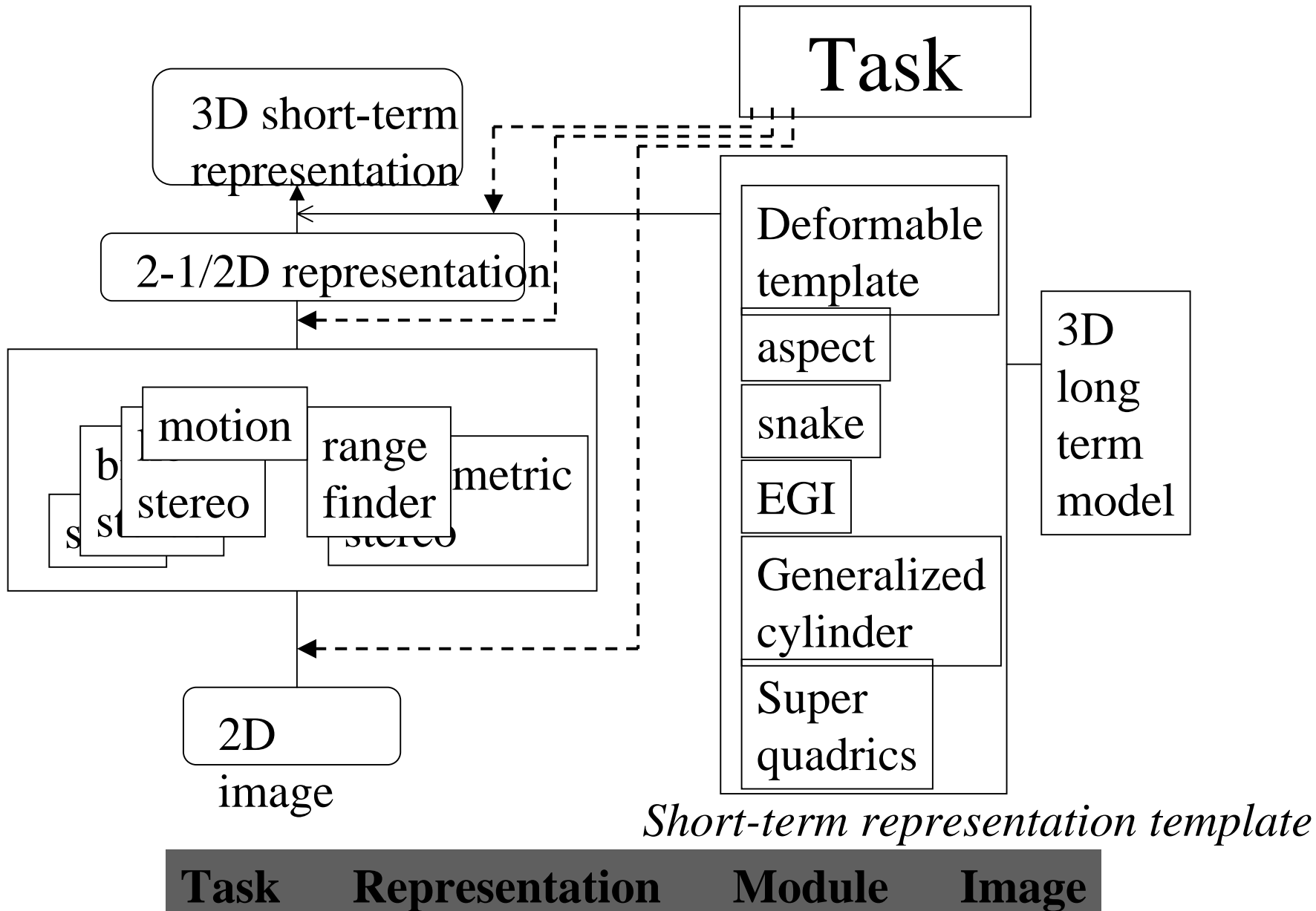


*Representation is arbitrary

Marr's school

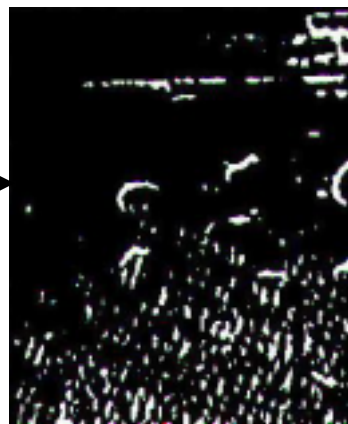


Task-oriented school





input scene



Segmentation result



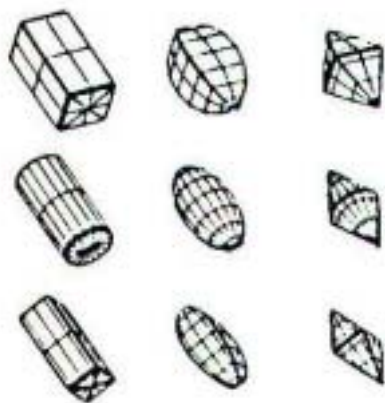
Superquadric representation of a rock



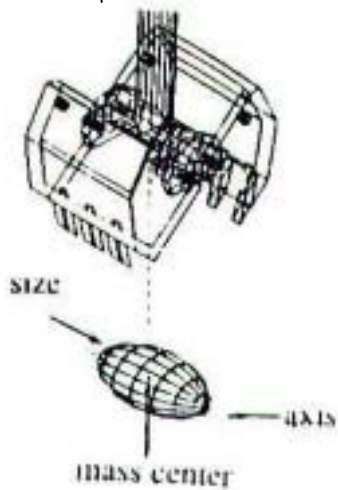
grasp plan



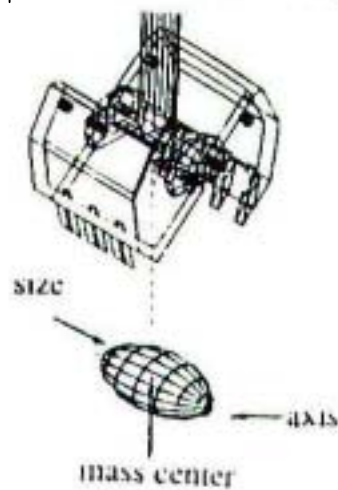
grasp execution



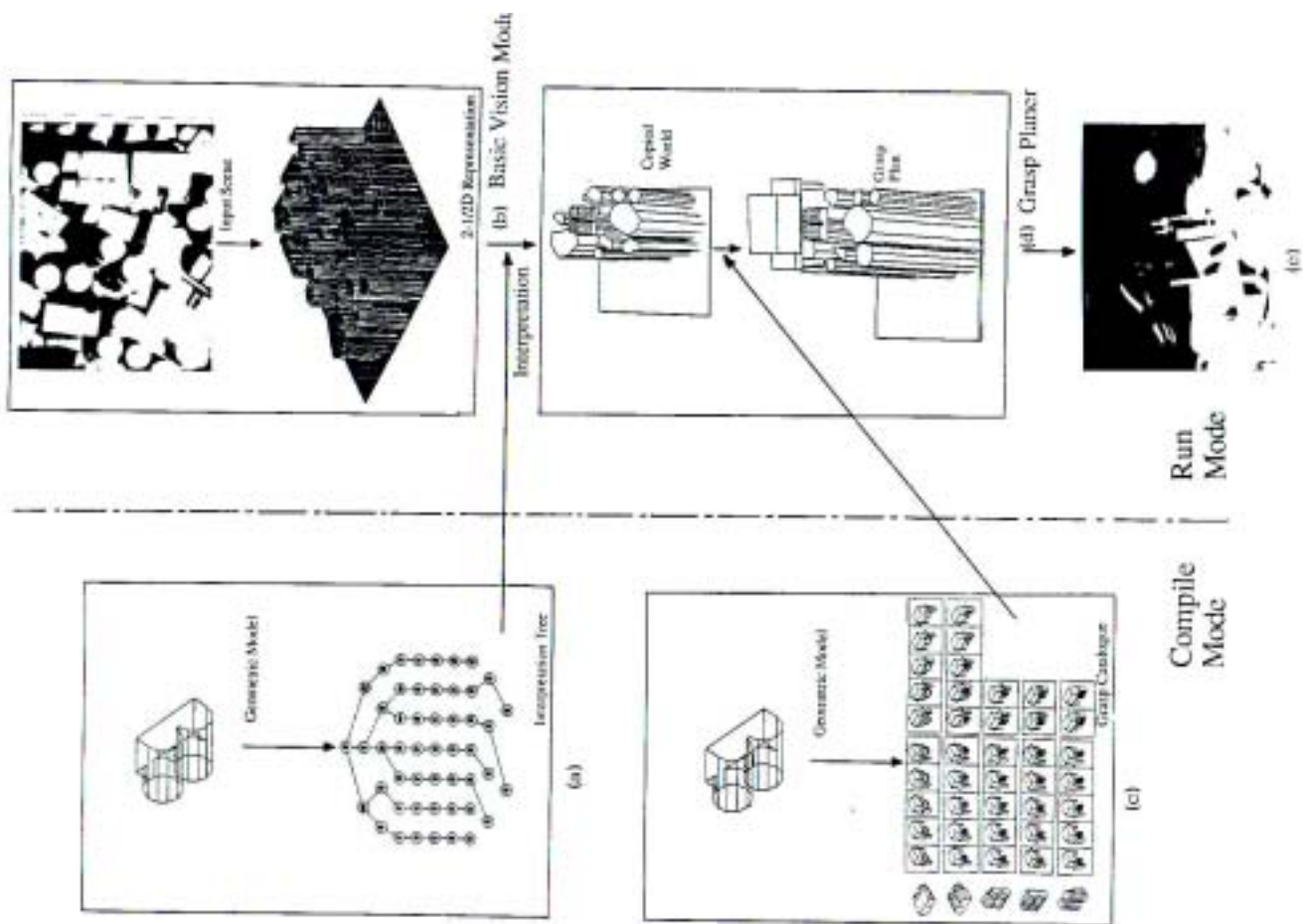
superquadric family



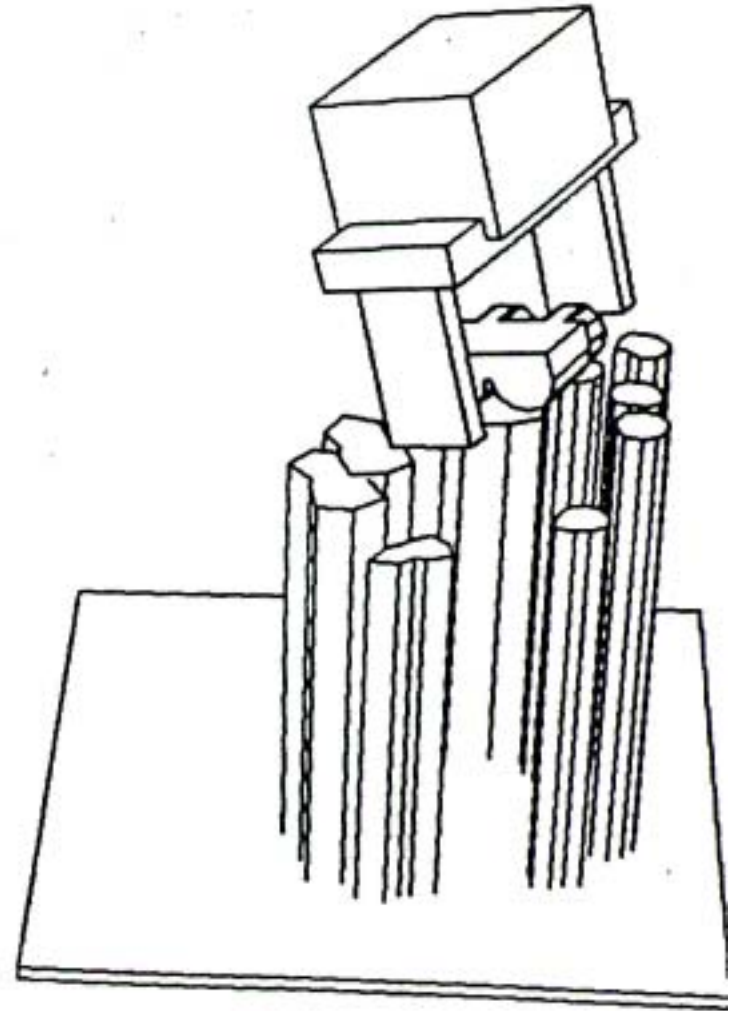
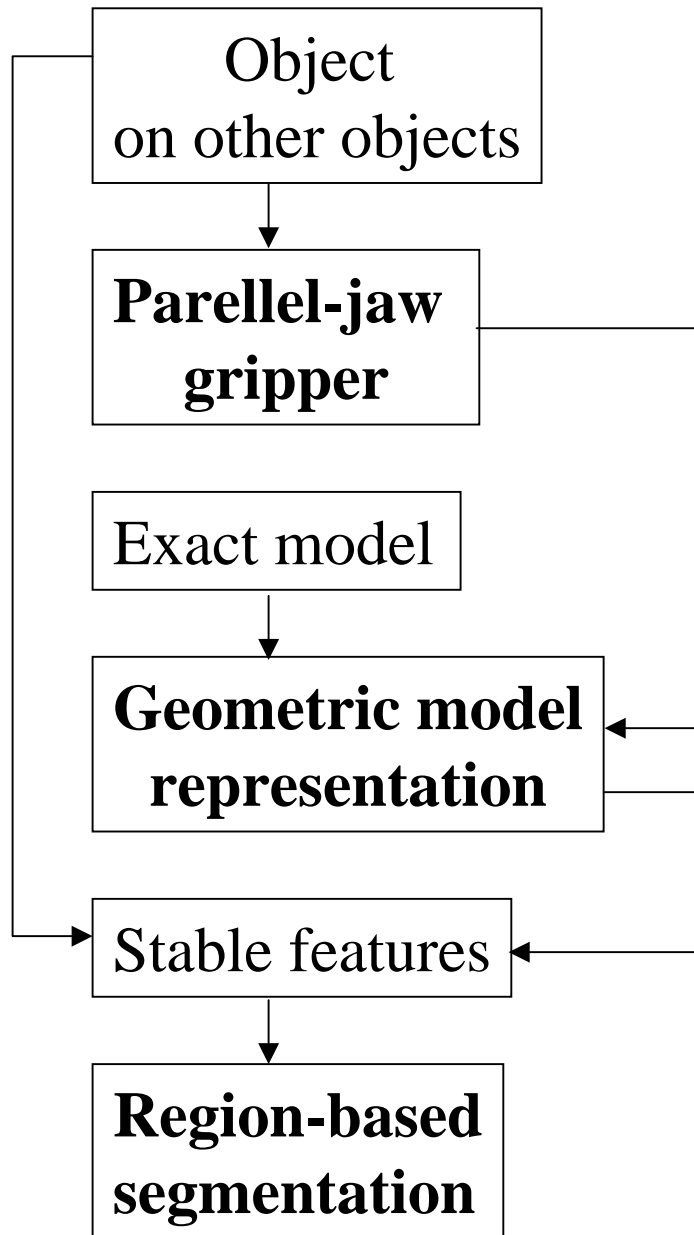
grasp strategy



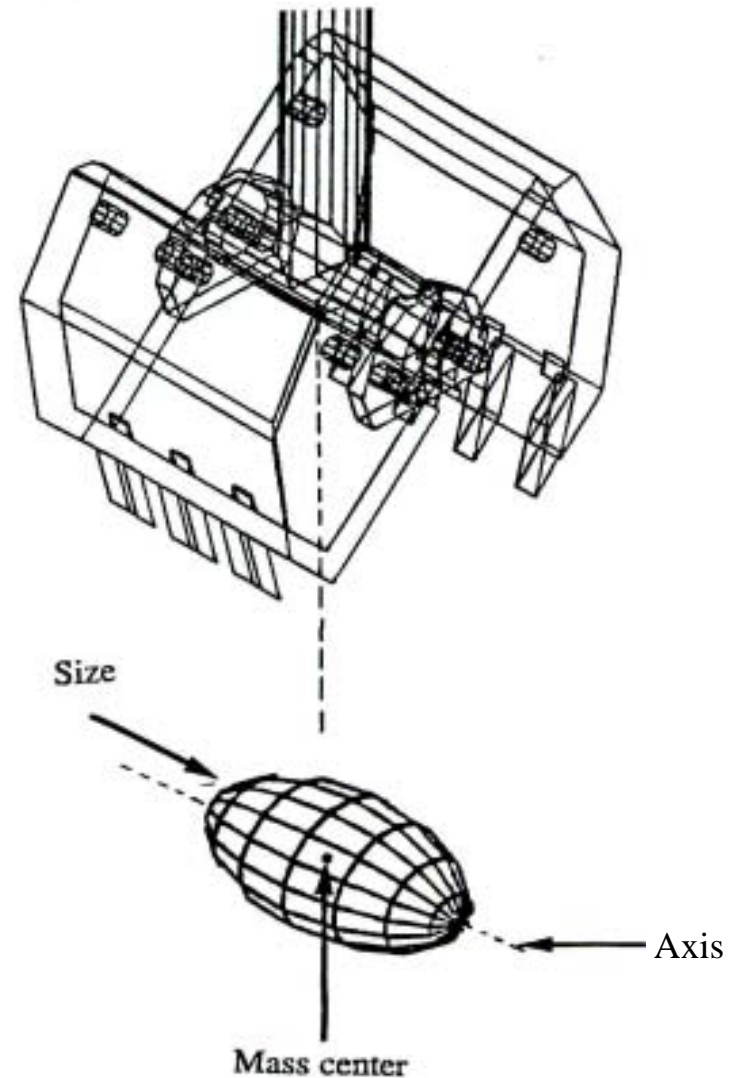
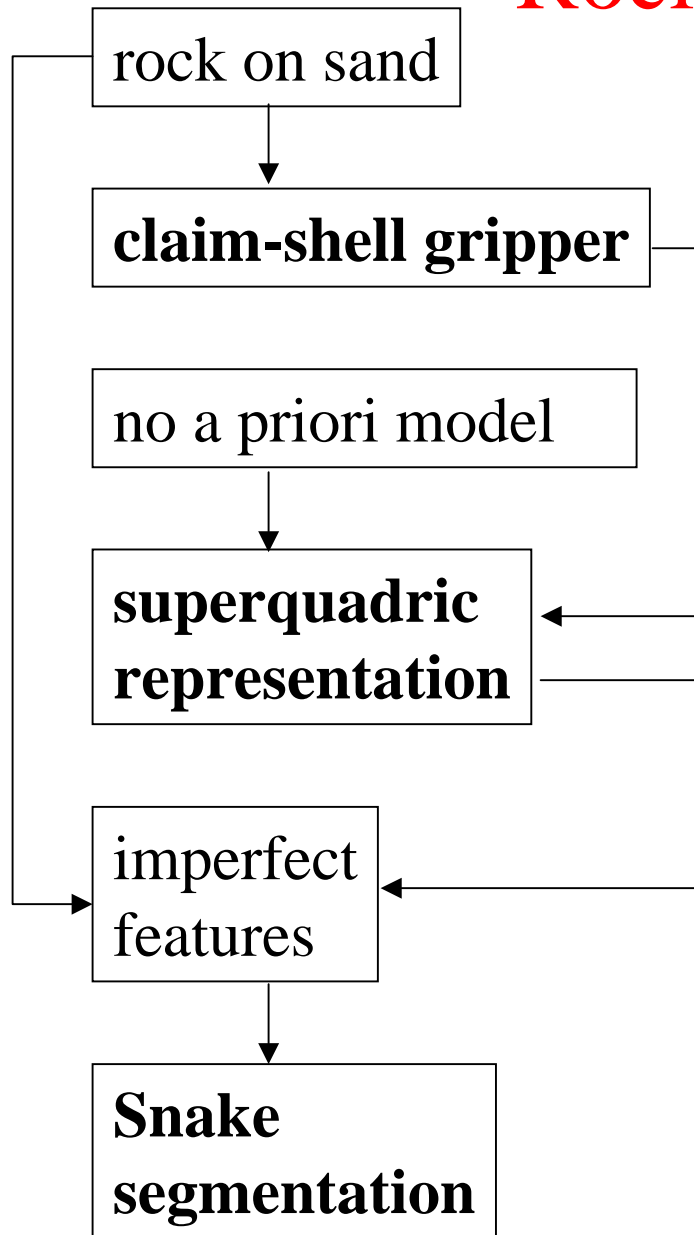
grasp strategy



Bin Picking



Rock Sampling



Taxonomy of tasks

TASK

WALK

• navigation

GRASP

• manipulation

SEARCH

• inspection

•••

to which
direction
to walk

where
to put
your foot

what
to grasp

how
to grasp

what
to search

how
to search

Taxonomy of grasping tasks



(a)



(b)



(c)




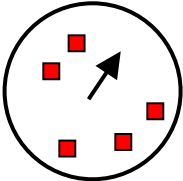

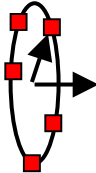

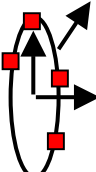

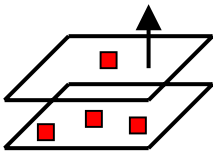

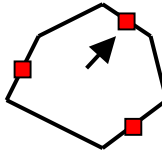

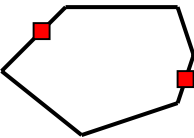
(d)



(e)



(f)

grasping type	required functional capabilities	representation
	 <p>~ <i>center</i> ~ <i>radius</i></p>	superquadrics
	 <p>~ <i>center</i> ~ <i>radius</i> ~ <i>axis direction</i></p>	generalized cylinder
	 <p>~ <i>center</i> ~ <i>radius</i> ~ <i>axis direction</i> ~ <i>pulling direction</i></p>	Superquadrics plus pulling direction
	 <p>orientation position of two planes width</p>	two parallel planes
	 <p>center radius</p>	cross-sectional shape
	 <p>position of points orientation</p>	two contact positions

Representation

- ◆ Requirement from a grasping strategy
- ◆ availability of a priori knowledge
- ◆ robustness of representation
- ◆ efficiency of representation
- ◆ computational cost to obtain
 - automatic selection of minimum cost feature set to obtain a representation

Representation Solids #2

We can classify representations into three categories: surface based, function based and volumetric based. This lecture covers surface-based and volumetric-based representations.

Surface-based representation

This method uses collections of faces to represent a solid: not necessarily all faces in the collections are visible. The method based on visible faces requires multiple representations for one object, but it is one easy method to use for model-based vision systems. The method based on all faces is easy to maintain because only one representation is necessary for one object, but the method is difficult to use because we have to consider which faces are visible from a given viewing direction.

Aspect representation -- An aspect is defined as topologically equivalent views of an object. An object is represented as collections of aspects.

Winged-edge representation -- A winged-edge representation is view independent, surface-based representation that contains lists of faces, edges, and vertices. Topological information between vertices, edges and faces is stored at edges. Numerical information is faces and vertices.

Volumetric-based representation

By using collections of primitive volumes with respect to its own coordinate system, this method represents a solid. This method is easy to store because we need only one representation for each object. This method, however, is difficult to handle because we have to determine visible shapes from a given viewing direction.

Oct-tree representation -- A solid is divided into octants, eight subdivided spatial cells. Each octant is labeled as either black (completely inside of the solid), white (completely outside of the solid), or gray (the boundary across the octant). Each gray octant is further subdivided into octant and labeled as either black, white or gray, recursively until necessary resolution is reached.

Constructive solid geometry representation -- A solid represented as a combination of set operations such as the union or intersection between primitive solids.

Summary

1 Surface based representation

- view dependent

 - » Oshima-shirai, aspect

- view independent

 - » Winged edge well-tellated sphere

2 Function based representation

3 Volume based representation

- oct-tree, cell-decomposition, CSG