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Criterion for Automatic Selection of the Most Suitable Maximum-Likelihood Thresholding Algorithm for Extracting Object from their Background in a Still Image

Geovanni Martinez
Image Processing and Computer Vision Research Lab (IPCV-LAB)
Escuela de Ingenieria Electrica, Universidad de Costa Rica
Codigo postal 2060, San Jose, Costa Rica
gmartin@pacuare.eie.ucr.ac.cr

Abstract

Three Maximum-Likelihood thresholding algorithms based on population mixture models are investigated and a criterion is introduced for automatic selection of the most suitable Maximum-Likelihood thresholding algorithm for extracting object from their background in an arbitrary still image. The most suitable algorithm is that whose population mixture model approximates better the probability density function of the intensity values. The probability density function is estimated from the histogram of the intensity values. The criterion was implemented and applied to real images with different illumination conditions. A subjective analysis of the experimental results showed that for each image, the proposed criterion was always able to select automatically from the three algorithms the one which delivers the best thresholding results.

1 Introduction

If objects do not touch each other, and if their intensity values are clearly distinct from background intensity values, thresholding is a suitable way for extracting object from their background. A variety of algorithms have been proposed in this regards [1, 2, 3]. Among the algorithms that estimate the threshold from the intensity values histogram of an image, the three Maximum-Likelihood thresholding algorithms based on population mixture models described in [4] are known as good algorithms and are widely used.

In those three Maximum-Likelihood thresholding algorithms the probability density function of the intensity values is described by a population mixture model [5, 6] under the assumption of normal distribution. In the first Maximum-Likelihood thresholding algorithm, the optimal threshold is estimated by maximizing the log conditional probability of the intensity values under the assumption of distinct means and a common variance in the population mixture model. This algorithm is equivalent to the Otsu's Algorithm described in [7]. In the second Maximum-Likelihood thresholding algorithm, the optimal threshold is estimated by maximizing the log joint probability of the intensity values under the assumption of distinct means and a common variance in the population mixture model. The third Maximum Likelihood thresholding algorithm is similar to the second one, but under the assumption of distinct means and distinct variances in the population mixture model. This algorithm is equivalent to the Kittler and Illingworth's algorithm described in [8].

In this contribution a criterion is presented for automatic selection of the most suitable Maximum-Likelihood algorithms for extracting object from their background in an arbitrary still image. To this end, that Maximum-Likelihood estimation algorithm, whose population mixture model approximates better the probability density function of the intensity values, will be considered to be the most suitable for extracting object from their background in a still image. The probability density function will be estimated from the histogram of the intensity values.

This paper is organized as follows. In section 2, the three Maximum-Likelihood thresholding algorithms are describe. In section 3, the criterion for selection of the most suitable Maximum-Likelihood thresholding algorithm is presented. In section 4 and in section 5, experimental results and the conclusions are given, respectively.

2 Maximum-Likelihood thresholding algorithms

Let us consider an intensity image whose pixels $I(x, y)$, $x = 0, ..., N - 1$, $y = 0..., M - 1$, assume discrete intensity values I in the interval [0, 255]. The distribution of the intensity values in the image can be displayed in the form of a histogram $h(I)$, $I = 1, .., 255$, which gives the frequency of occurrence of each intensity value in the image. The corresponding probability density function of the intensity values can be obtained by normalizing the histogram of the intensity values in the form of $p(I) = h(I)/(N \cdot M)$, where $N \cdot M$ is the total number of pixels in the image.

Now suppose that we are classifying the pixels into two classes C_1 and C_2 (background and objects, or viceversa) by threshold at value k. Here C_1 denotes pixels with intensity values $[0, ..., k]$ and C_2 denotes pixels with intensity values $[k + 1, ..., 255]$.

Now let describe the probability density function $p(I)$ by a population mixture model consisting of the sum of two weighted conditional Gaussian probability density functions with means m_1 and m_2, variances σ_1^2 and σ_2^2 and weights c_1 and c_2, where $c_1 + c_2 = 1$:

$$p(I) = c_1 \cdot p_1(I/C_1) + c_2 \cdot p_2(I/C_2)$$

$$p(I) = \sum_{j=1}^{2} \frac{c_j}{\sqrt[2]{2\pi\sigma_j^2}} e^{-\frac{(I-m_j)^2}{2\sigma_j^2}}. \quad (1)$$

The statistical parameters of $p_1(I/C_1)$ and $p_2(I/C_2)$ can be estimated from $p(I)$ depending on the threshold at value k as follows:

$$c_1(k) = \sum_{I=0}^{k} p(I) \quad (2)$$

$$c_2(k) = \sum_{I=k+1}^{255} p(I) \quad (3)$$

$$m_1(k) = \frac{1}{c_1(k)} \sum_{I=0}^{k} I \cdot p(I) \quad (4)$$

$$m_2(k) = \frac{1}{c_2(k)} \sum_{I=k+1}^{255} I \cdot p(I) \quad (5)$$

$$\sigma_1^2(k) = \frac{1}{c_1(k)} \sum_{I=0}^{k} (I - m_1)^2 \cdot p(I) \quad (6)$$

$$\sigma_2^2(k) = \frac{1}{c_2(k)} \sum_{I=k+1}^{255} (I - m_2)^2 \cdot p(I) \quad (7)$$

Let $S(x,y) = [S(x,y)_1, S(x,y)_2]^T$ be a two-dimensional vector with a single zero and a single 1, where the position of the 1 indicates which class the pixel $I(x,y)$ belongs to. Then the conditional probability density function given $S(x,y)$ is:

$$p(I(x,y)|S(x,y)) = \sum_{j=1}^{2} S_j(x,y) \cdot p(I(x,y)|C_j) = \prod_{j=1}^{2} p(I(x,y)|C_j)^{S_j(x,y)} \quad (8)$$

The marginal probability density function of the vectors $S(x,y)$ and the joint probability density function can be written as:

$$p(S(x,y)) = \prod_{j=1}^{2} c_j^{S_j(x,y)}, \quad (9)$$

$$p(I(x,y), S(x,y)) = p(S(x,y)) \cdot p(I(x,y)|S(x,y)) = \prod_{j=1}^{2} [c_j \cdot p(I(x,y)|C_j)]^{S_j(x,y)}. \quad (10)$$

Now suppose that the pairs $(I(x,y), S(x,y))$ are statistical independent. The conditional probability density and the joint probability density of the values of I given the values of S are given by:

$$p(I(0,0),...,I(N-1,M-1)|S(0,0),...,S(N-1,M-1)) = \prod_{x=0}^{N-1} \prod_{y=0}^{M-1} \prod_{j=1}^{2} [p(I(x,y)|C_j)]^{S_j(x,y)}, \quad (11)$$

$$p(I(0,0),...,I(N-1,M-1), S(0,0),...,S(N-1,M-1)) = \prod_{x=0}^{N-1} \prod_{y=0}^{M-1} \prod_{j=1}^{2} [c_j \cdot p(I(x,y)|C_j)]^{S_j(x,y)}. \quad (12)$$

If the Gaussian probability functions $p_1(I/C_1)$ and p_2 (I/C_2) have different means $m_1(k)$ and $m_2(k)$, and a common variance given by:

$$\sigma_W^2(k) = c_1(k) \cdot \sigma_1^2(k) + c_2(k) \cdot \sigma_2^2(k), \quad (13)$$

then the following likelihoods result from the log of the Eq. 11 and the log of the Eq. 12:

$$L_1(k) = -\frac{N}{2} \cdot \log(2\pi) - \frac{N}{2} \log(\sigma_W^2(k)) - \frac{N}{2}, \quad (14)$$

$$L_2(k) = N \cdot \sum_{j=1}^{2} c_j(k) \cdot \log(c_j(k)) + L_1(k). \quad (15)$$

In the first Maximum-Likelihood thresholding algorithm that threshold k which maximizes the likelihood function $L_1(k)$ is considered to be the optimal threshold. This algorithm is equivalent to the Otsu's Algorithm described in [7]. In the second Maximum-Likelihood thresholding algorithm that threshold k which maximizes the likelihood function $L_2(k)$ is considered to be the optimal threshold.

If the Gaussian probability functions $p_1(I/C_1)$ and p_2 (I/C_2) have different means $m_1(k)$ and $m_2(k)$, and different variances $\sigma_1^2(k)$ and $\sigma_2^2(k)$, then the following likelihood results from the log of the Eq. 11:

$$L_3(k) = N \cdot \sum_{j=1}^{2} c_j(k) \cdot \log(c_j(k)) - \frac{N}{2} \log(2\pi) - + \frac{N}{2} \cdot \sum_{j=1}^{2} c_j(k) \cdot \log(\sigma_j^2(k)) - \frac{N}{2}. \quad (16)$$

In the third Maximum-Likelihood thresholding algorithm that threshold k which maximizes the likelihood function $L_3(k)$ is considered to be the optimal threshold. This algorithm is equivalent to the Kittler and Illingworth's algorithm described in [8].

3 Selection criterion

Let assume that k_1, k_2 and k_3 are the optimal thresholds obtained maximizing Eq. 14, Eq. 15 and Eq. 16. Using those three thresholds the following three population mixture models can be estimated:

$$p_{k_1}(I) = \sum_{j=1}^{2} \frac{c_j(k_1)}{\sqrt[2]{2\pi\sigma_j^2(k_1)}} e^{-\frac{(I-m_j(k_1))^2}{2\sigma_j^2(k_1)}}, \qquad (17)$$

$$p_{k_2}(I) = \sum_{j=1}^{2} \frac{c_j(k_2)}{\sqrt[2]{2\pi\sigma_j^2(k_2)}} e^{-\frac{(I-m_j(k_2))^2}{2\sigma_j^2(k_2)}}, \qquad (18)$$

$$p_{k_3}(I) = \sum_{j=1}^{2} \frac{c_j(k_3)}{\sqrt[2]{2\pi\sigma_j^2(k_3)}} e^{-\frac{(I-m_j(k_3))^2}{2\sigma_j^2(k_3)}}. \qquad (19)$$

Then the mean square error between the probability density function $p(I)$ estimated from the histogram $h(I)$ and each one of the above estimated population mixture models are computed as follows:

$$mse(k_1) = \frac{1}{256}\sum_{I=0}^{255} \left[p_{k_1}(I) - p(I)\right]^2, \qquad (20)$$

$$mse(k_2) = \frac{1}{256}\sum_{I=0}^{255} \left[p_{k_2}(I) - p(I)\right]^2, \qquad (21)$$

$$mse(k_3) = \frac{1}{256}\sum_{I=0}^{255} \left[p_{k_3}(I) - p(I)\right]^2. \qquad (22)$$

The most suitable threshold k_s for extracting object from their background in an arbitrary image is that one that minimizes the mean square error:

$$mse(k_s) \leq mse(k), k = k_1, k_2, k_3. \qquad (23)$$

4 Experimental results

We have implemented the three different Maximum-Likelihood thresholding algorithms and the proposed selection criterion in the programming language C, under the operating system XP and performed a number of experiments on 189 real intensity images with different illumination conditions on a 2.2 Ghz desktop with 1.0 GB RAM. The average processing time per image was 0.041 $seconds$. Due to the lack of space, we present just the experimental results obtained from four real images only.

Figs. 1(a), 2(a), 3(a) and 4(a) depict the original intensity images. Figs. 1(b), 2(b), 3(b) and 4(b) depict the resulted binary images after thresholding using the first Maximum-Likelihood thresholding algorithm (Otsu's algorthm). Figs. 1(c), 2(c), 3(c) and 4(c) depict the resulted binary images after thresholding using the second Maximum-Likelihood thresholding algorithm. Figs. 1(d), 2(d), 3(d) and 4(d) depict the resulted binary images after thresholding using the third Maximum-Likelihood thresholding algorithm (Kittler and Illingworth's algorithm). Figs. 1(e), 2(e), 3(e) and 4(e) depict the resulted binary images after thresholding using the selected Maximum-Likelihood thresholding algorithm. For Fig. 1 and Fig. 3 the second Maximum-Likelihood Algorithm was selected. For Fig. 2 and Fig. 4 the third Maximum-Likelihood Algorithm was

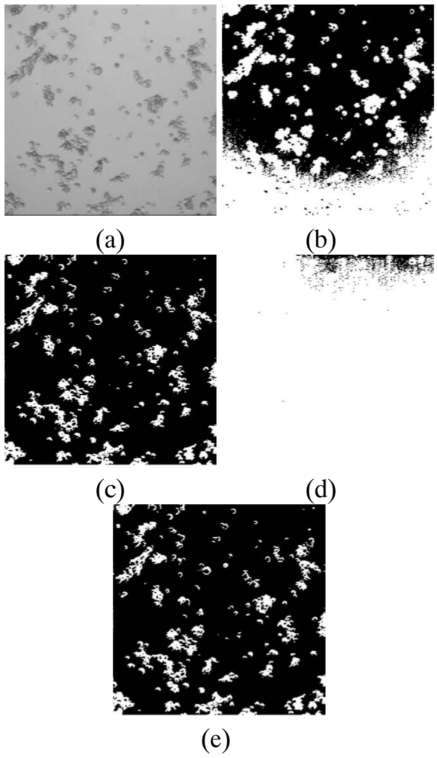


Figure 1. (a) Original cell image. (b) Binary image obtained with the 1st ML thresholding algorithm. (c) Binary image obtained with the 2nd ML algorithm. (d) Binary image obtained with the 3rd ML algorithm. (e) Binary image obtained with the selected ML algorithm (in this example the second ML algorithm was selected).

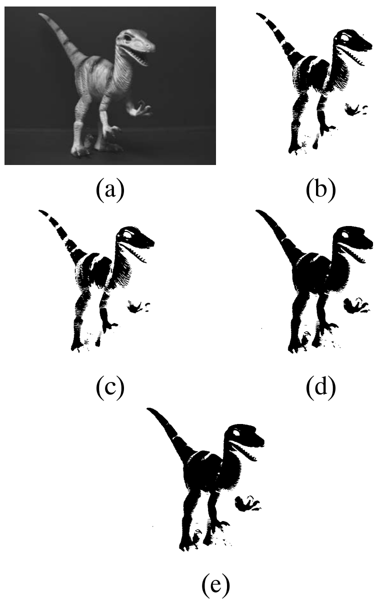
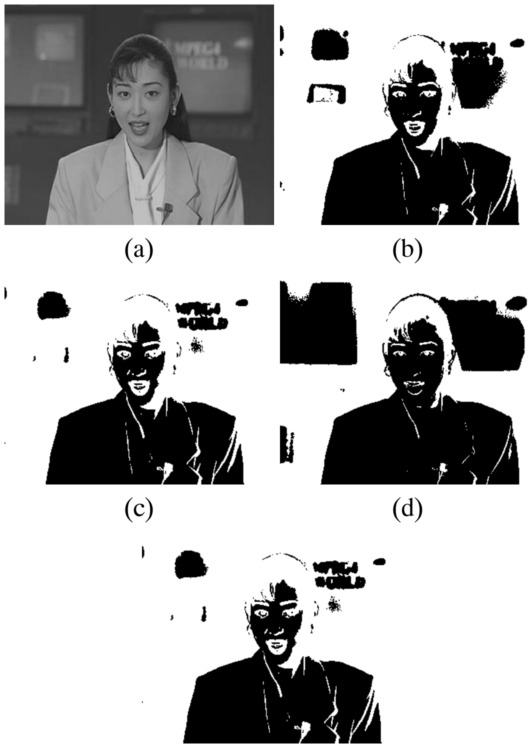
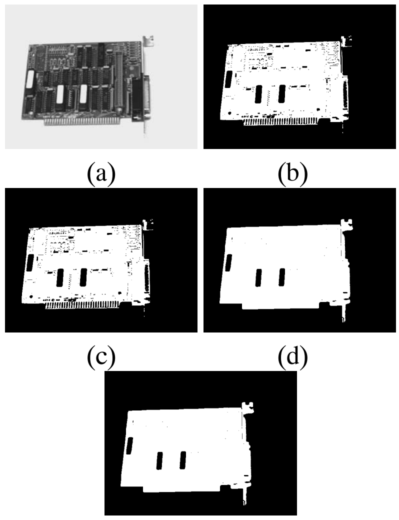


Figure 2. (a) Original dinosaur image. (b) Binary image obtained with the 1st ML thresholding algorithm. (c) Binary image obtained with the 2nd ML algorithm. (d) Binary image obtained with the 3rd ML algorithm. (e) Binary image obtained with the selected ML algorithm (in this example the third ML algorithm was selected).



(a) (b) (c) (d) (e)

Figure 3. (a) Original Akiyo image. (b) Binary image obtained with the 1st ML thresholding algorithm. (c) Binary image obtained with the 2nd ML algorithm. (d) Binary image obtained with the 3rd ML algorithm. (e) Binary image obtained with the selected ML algorithm (in this example the second ML algorithm was selected).



(a) (b) (c) (d) (e)

Figure 4. (a) Original card image. (b) Binary image obtained with the 1st ML thresholding algorithm. (c) Binary image obtained with the 2nd ML algorithm. (d) Binary image obtained with the 3rd ML algorithm. (e) Binary image obtained with the selected ML algorithm (in this example the third ML algorithm was selected).

selected. A subjective analysis of the above experimental results show that for each image, the proposed criterion was always able to select automatically from the three algorithms the one which delivers the best subjective thresholding results.

5 Conclusions

In this contribution a criterion for automatic selection of the most suitable Maximum-Likelihood algorithm for thresholding of an arbitrary still image is presented. First, three Maximum-Likelihood thresholding algorithms are applied to the image. Then, for each Maximum-Likelihood thresholding algorithm, the mean square error between the estimated population mixture model and the probability density function estimated from the intensity values histogram is computed. Finally, that algorithm, whose estimated population mixture model produces the smallest mean square error, is supposed to be the most suitable for background extraction of the arbitrary still image. A subjective analysis of the experimental results showed that for each image, the proposed criterion was always able to select automatically from the three algorithms the one which delivers the best thresholding results.

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6 References

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